System on a Chip

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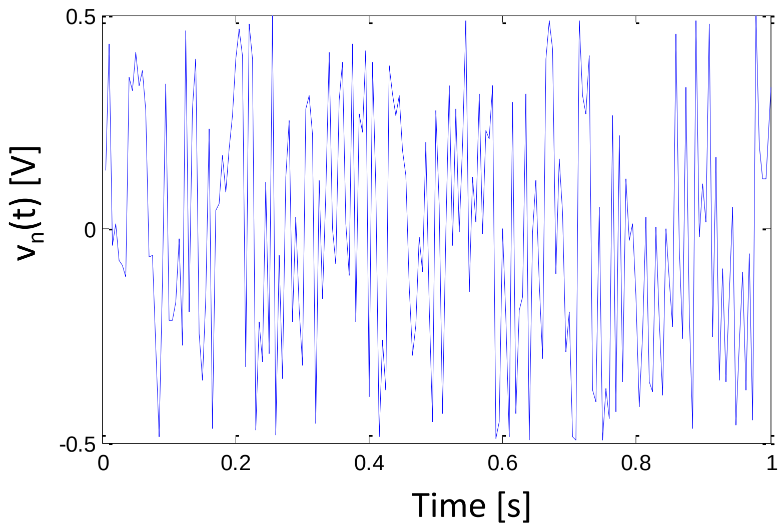
Lecture 9: Fundamentals of Noise

- Noise
  - Background/Theory
  - Noise in Devices

Background

- Any physical signal consists of a desired signal and an unwanted component
- Any random unwanted component is called NOISE
- There are two noise types:
  - Interference noise originates from unwanted interaction between the circuit and the outside world or another part of the circuit. Possible solutions: shielding, guard rings, ground planes, separate ground and power lines.
  - Inherent noise is produced by the circuit components themselves. It can never be eliminated but its influence on the circuit may be reduced.
- Inherent noise generators: Resistors, diodes, BJT, MOSFET. (Capacitors are noiseless!)
- Noise in generated by small current and voltage fluctuations. It is basically due to the fact that electrical charge is carried in discrete amounts equal to the electron charge.

Mean and RMS



Example of a random signal

Mean value:

$$\overline{v_n} = \frac{1}{T}\int_0^T v_n(t)\ dt = 0$$ for T large enough

Physical noise signals have a mean of zero

Root mean square value:

$$\overline{v_{n(RMS)}} = \sqrt{\frac{1}{T}\int_0^T v_n(t)^2\ dt} \neq 0$$ constant for T large enough

- The instantaneous value of a noise signal is undetermined.
- To characterise a noise signal, the mean value and root mean square value are used
- The standard deviation is equal to the RMS; the variance is equal to the mean square value (=RMS^2)

Mean Square Value

Mean square value:

$$\overline{{v_{n(RMS)}}^2} = \frac{1}{T}\int_0^T v_n(t)^2 \; dt \neq 0$$

- Mean square value is a measure for the normalised noise power of the signal.
- The power dissipation by a 1Ω resistor with a dc voltage of aV applied across it is equivalent to a noise source with a RMS voltage of aV.

- Most physical noise generators have a mean value of zero, here only such noise sources are considered.
- All equations are equivalent for current noise generators.

Signal to Noise Ratio (SNR)

SNR is defined as:

$$SNR = 10\log\left[\frac{signal\ power}{noise\ power}\right]$$

SNR can be calculated at any node in a circuit. Consider a signal $v_x(t)$ that has a normalized signal power of $V^2_{x(RMS)}$ and a normalised noise power of $V^2_{n(RMS)}$ the SNR is given by:

$$SNR = 10\log\left[\frac{V^2_{x(RMS)}}{V^2_{n(RMS)}}\right] = 20\log\left[\frac{V_{x(RMS)}}{V_{n(RMS)}}\right]$$

- SNR are usually measured in dB. NOTE: Since power levels are used, dB is calculated by using an multiplicand of 10.
- dBm is often used as well. All power levels are referenced to 1mW, i.e. 1mW=0dBm, 1mW=-30dBm
- If voltage is measured – reference level to equivalent power dissipated if voltage is applied to 50Ω (or sometimes 75 Ω) resistor

(SNR)dbM Example

- Find rms voltage of 0 dBm signal (50Ω reference)
- What is level in dBm of a 2 volt rms signal?

- 0 dBm signal (50Ω reference) implies

$$V_{(rms)} = \sqrt{(50\Omega) \times 1mW} = 0.2236$$

- Thus, a 2 volt (rms) signal corresponds to

$$20 \times \log\left(\frac{2.0}{0.2236}\right) = 19 \text{ dBm}$$

Noise Summation

- Typically, a circuit contains many noise generators. For circuit analysis we usually sum all noise sources into a single one (either at the output or the input of the circuit) and treat the circuit as noiseless.
- Summing noise sources is done by adding their mean square values:

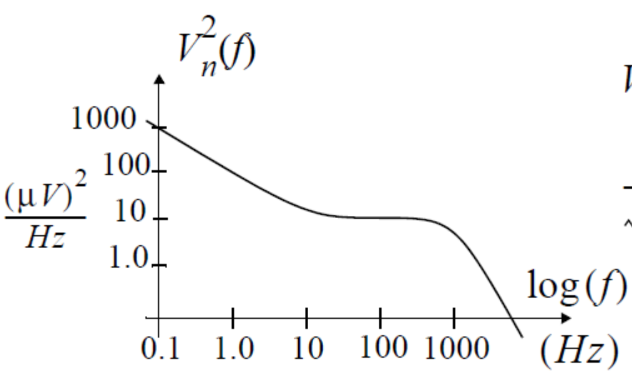
$$V_{n(RMS)}^2 = V_{n1(RMS)}^2 + V_{n2(RMS)}^2 \dots + V_{nj(RMS)}^2$$

Example:

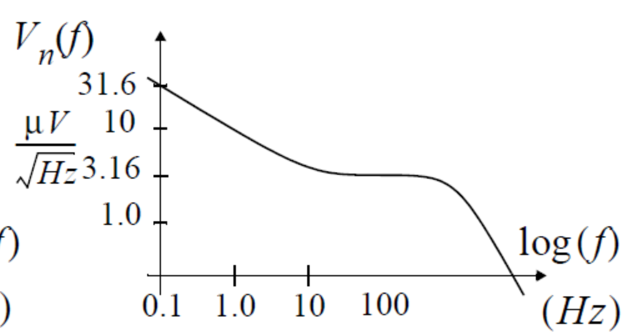
What is the total output RMS value of two uncorrelated noise sources with $V_{n1(RMS)}$=10μV and $V_{n2(RMS)}$=5μV? If the total RMS value is required to be less than 10mV, how much should $V_{n1(RMS)}$ be reduced while $V_{n2(RMS)}$ remains constant?

Noise Spectral Density

- Noise signal are always spread out over the frequency spectrum
- Imagine passing a noise signal through a narrowband tuned filter and measuring the mean square output in a frequency band (e.g. 1Hz)
- The mean squared value of a random noise signal at a single precise frequency is zero.
- Other common measure is the square root spectral density in units of V/√Hz
- S(f) is the autocorrelation function of the time domain signal $v_n(t)$ (Wiener-Khinchin theorem).



Spectral Density



Root-Spectral Density

Total noise power:

$$\overline{v_{n(RMS)}}^2 = \int_0^\infty S(f)\, df$$

In band noise power:

$$\overline{v_{n(RMS)}}^2 = \int_0^B S(f)\, df$$

Spectral Density

Spectral Density $V_n^2(f)$

- Average normalized power over a 1 hertz bandwidth
- Units are volts-squared/hertz

Root-Spectral Density $V_n(f)$

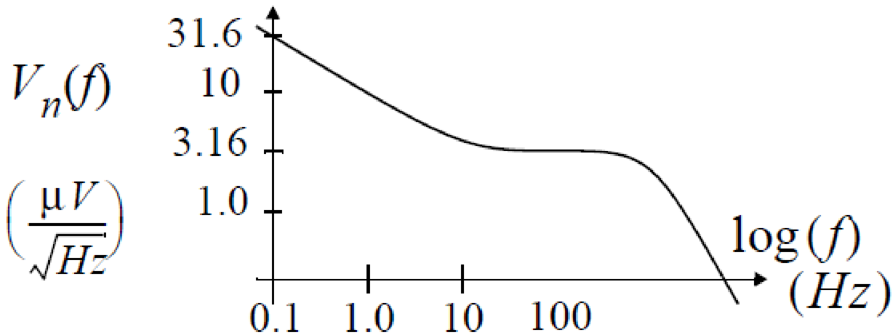
- Square root of vertical axis (freq axis unchanged)
- Units are volts/root-hertz (i.e. $V / \sqrt{Hz}$).

Total Power

$$V_{n(rms)}^2 = \int_0^\infty V_n^2(f) df \tag{15}$$

- Above is a one-sided definition (i.e. all power at positive frequencies)

Root Spectral Density



- Around 100 Hz, $V_n(f) = \sqrt{10}\ \mu\text{V}/\sqrt{\text{Hz}}$
- If measurement used RBW $= 30$ Hz, measured rms

$$\sqrt{10} \times \sqrt{30} = \sqrt{300}\ \mu\text{V}$$

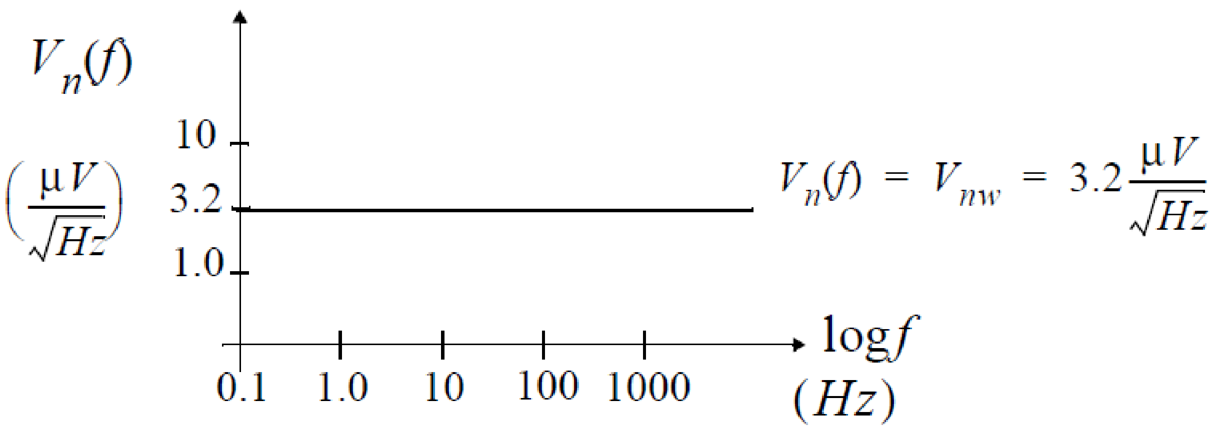
- If measurement used RBW $= 0.1$ Hz, measured rms

$$\sqrt{10} \times \sqrt{0.1} = 1\ \mu\text{V}$$

Types of Noise

1. White Noise

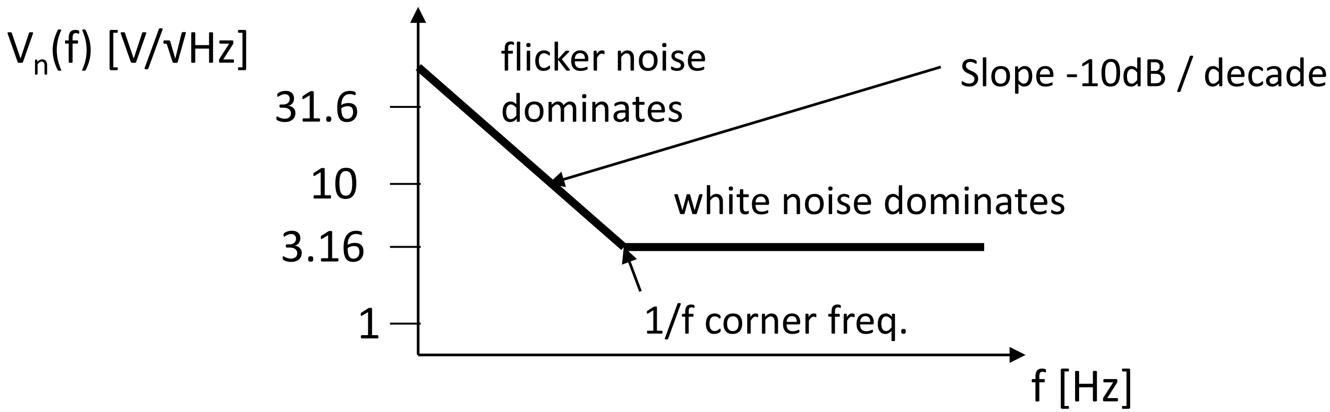
- White noise has a flat (or constant) spectral density, i.e. spectral density = const.
- White noise is produced by thermal noise generators (or Johnson, Boltzman).
- Examples are resistors, BJT's and MOS transistors. These noise generators can be assumed to be white up to a few THz.



Types of Noise

2. Flicker or 1/f noise

- The spectral density is proportional to 1/f: $v_n^2(f)=k^2_f / f$
- Root spectral density is $v_n(f)=k_f/(sqrt(f)$
- Falls off at -10db/decade due to sqrt(f)
- Flicker noise is important in MOS transistors, especially at low frequencies
- MOS transistors have both flicker noise and white noise



Filtered Noise

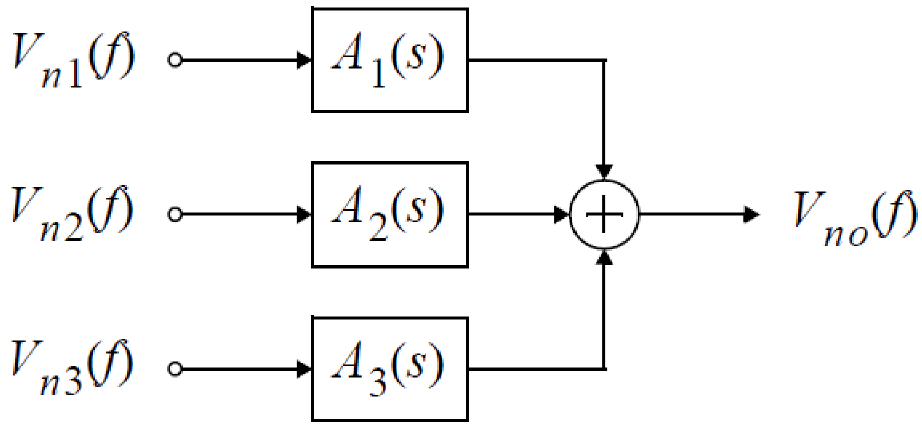
- A noise signal is shaped by a transfer function $A(j2\pi f)$.
- The spectral density at the output due to the noise is given by: $S_o(f)=|A(j2\pi f)|^2 S_i(f)$
- It is shaped only by the magnitude of the transfer function, not by its phase.
- The total output mean square value is given by:

$$\overline{v_{no(RMS)}{}^2} = \int_0^{\infty} \left|A(j2\pi f)\right|^2 S(f)\, df$$

Filtered Noise

- **Example**
- What is the total RMS value of a white noise signal $V_{ni}(f)$ with a root spectral density of $20nV/\sqrt{Hz}$ in a bandwidth from dc to 100kHz? What is the total noise RMS value if it filtered by a RC filter ($R=1k\Omega$, $C=0.159\mu F$) which is assumed noiseless?

Sum of Filtered Noise



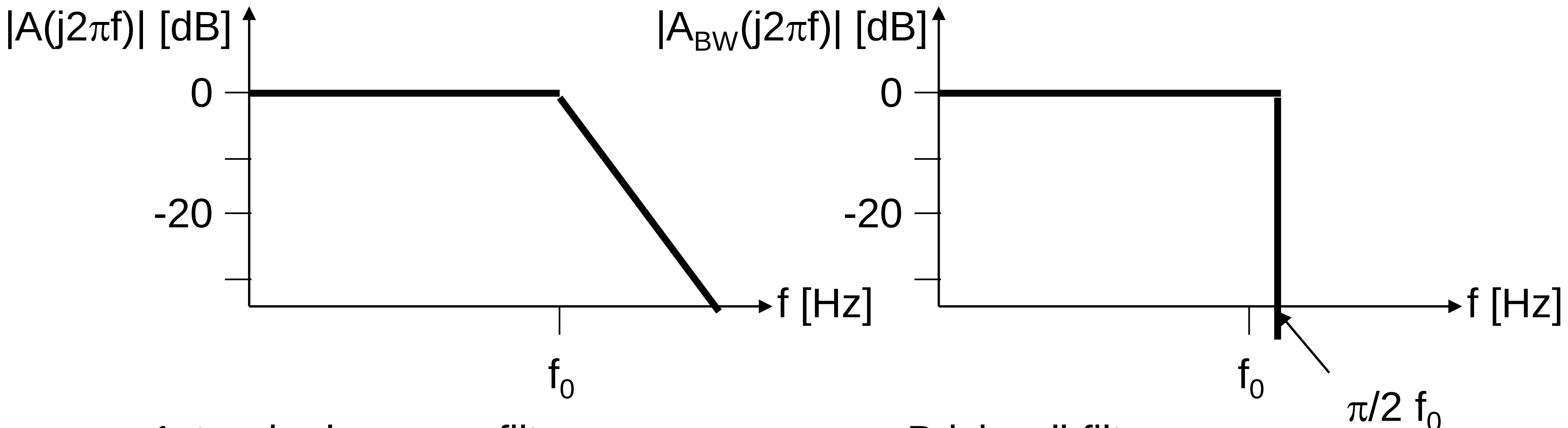
uncorrelated noise sources

- If filter inputs are uncorrelated, filter outputs are also uncorrelated
- Can show

$$V_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f)$$

Equivalent Noise Bandwidth

- The total RMS noise power from dc to infinity of a signal at the output any practical low pass filter is finite.
- An equivalent ideal brickwall filter can be found that has the same V_n^2(RMS) as a practical low pass filter (the peak gain of the ideal and real filter are the same).
- The bandwidth of this brickwall filter is the equivalent noise bandwidth.



1st order low pass filter having a noise bandwidth of $\pi/2\ f_0$

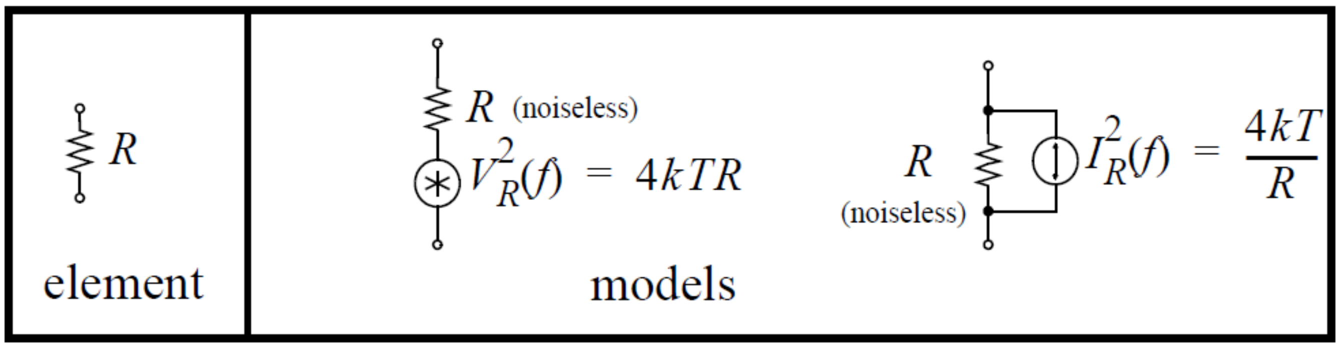
Brickwall filter

Noise Models for Circuit Elements

- Three main sources of noise:
- **Thermal Noise**
  - Due to thermal excitation of charge carriers
  - Appears as white spectral density
- **Shot Noise**
  - Due to dc bias current being pulses of carriers
  - Dependent of dc bias current and is white
- **Flicker Noise**
  - Due to traps in semiconductors
  - Has a 1/f spectral density
  - Significant in MOS transistors at low frequencies

Resistor Noise

- Thermal noise — white spectral density
- k is Boltzmann's constant = 1.38x1e-23 JK^{-1}
- T is the temperature in degrees Kelvin

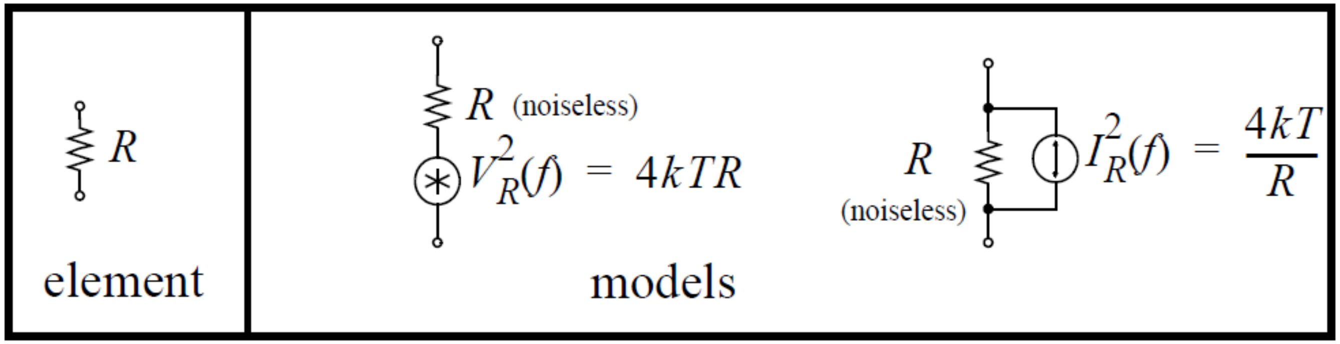


- Alternatively:

$$V_R(f) = \sqrt{\frac{R}{1k}} \times 4.06\ nV/\sqrt{Hz} \quad \text{for } 27^\circ C$$

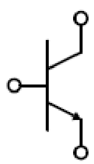
Diode Noise

- Shot noise — white spectral density
- q is the charge of an electron = 1.6 x 1e-19 C
- I_D is the dc bias current through the diode

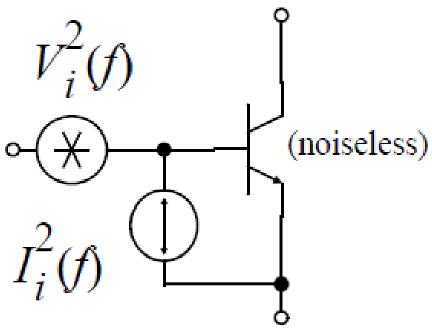


Bipolar Transistors

- Shot noise of collector and base currents
- Flicker noise due to base current
- Thermal noise due to base resistance
- $V_i(f)$ has base resistance thermal noise plus collector shot noise referred back
- $I_i(f)$ has base shot noise, base flicker noise plus collector shot noise referred back



(active-region)
element



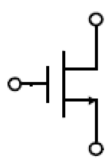
model

$$V_i^2(f) = 4kT\left(r_b + \frac{1}{2g_m}\right)$$

$$I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{|\beta(f)|^2}\right)$$

MOSFETS

- Flicker noise at gate
- Thermal noise in channel



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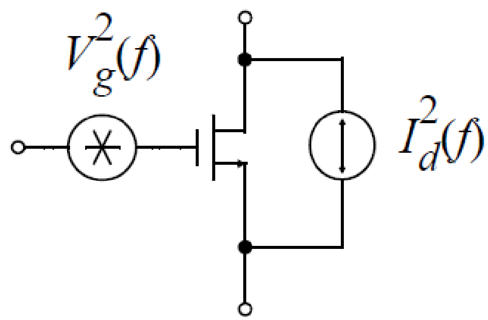


$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$$

model

MOSFETS 1/f Noise

$$V_g^2(f) = \frac{K}{WLC_{ox}f} \tag{36}$$

- K dependent on device characteristics, varies widely.
- W & L — Transistor's width and length
- C_{ox} — gate-capacitance/unit area

- ***Flicker noise is inversely proportional to the transistor area***, WL.
- 1/f noise is extremely important in MOSFET circuits as it can dominate at low-frequencies
- Typically p-channel transistors have less noise since holes are less likely to be trapped.

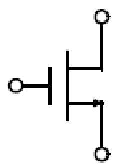
MOSFETS Thermal Noise

- Due to resistive nature of channel
- In triode region, noise would be $I_d^2(f) = (4kT)/r_{ds}$ where r_{ds} is the channel resistance
- In active region, channel is not homogeneous and total noise is found by integration

$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m \tag{37}$$

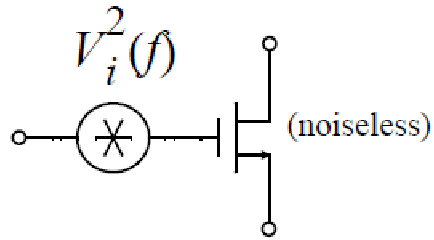
for the case $V_{DS} = V_{GS} - V_T$

Low Moderate Frequency MOSFET



(active-region)

element

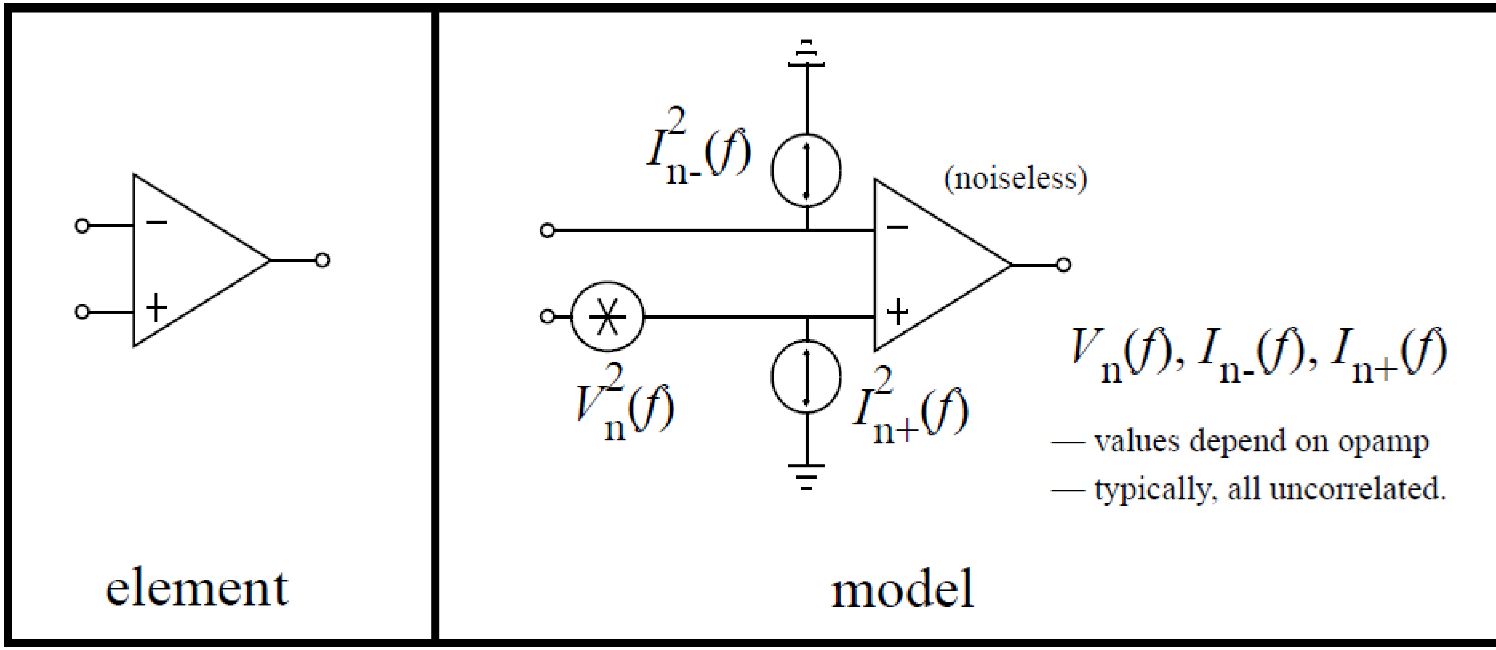


$$V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$$

model

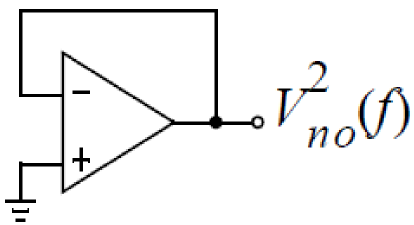
- Can lump thermal noise plus flicker noise as an input voltage noise source at low to moderate frequencies.
- At high frequencies, gate current can be appreciable due to capacitive coupling.

Opamps



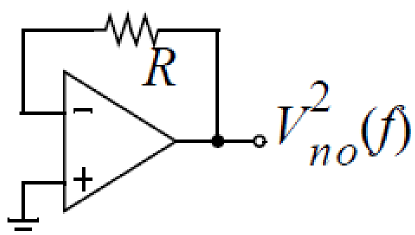
- Modelled as 3 uncorrelated input-referred noise sources.
- Current sources often ignored in MOSFET opamps

Opamps



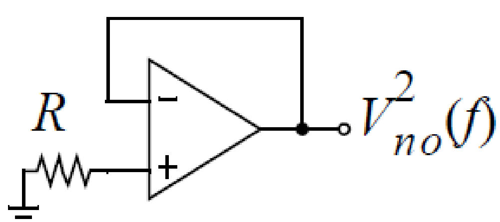
$V_n(f)$ ignored $\Rightarrow V^2_{no} = 0$

Actual $V^2_{no} = V^2_n$



$I_{n-}(f)$ ignored $\Rightarrow V^2_{no} = V^2_n$

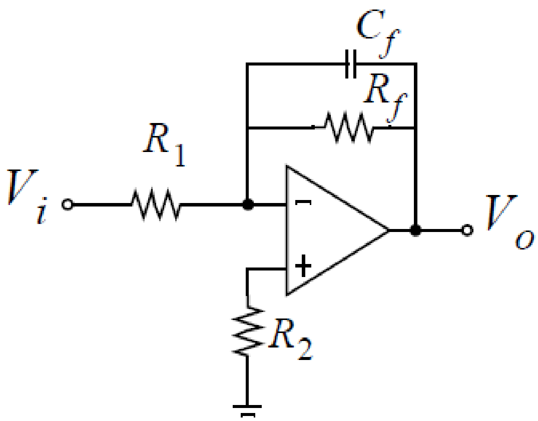
Actual $V^2_{no} = V^2_n + (I_{n-}R)^2$



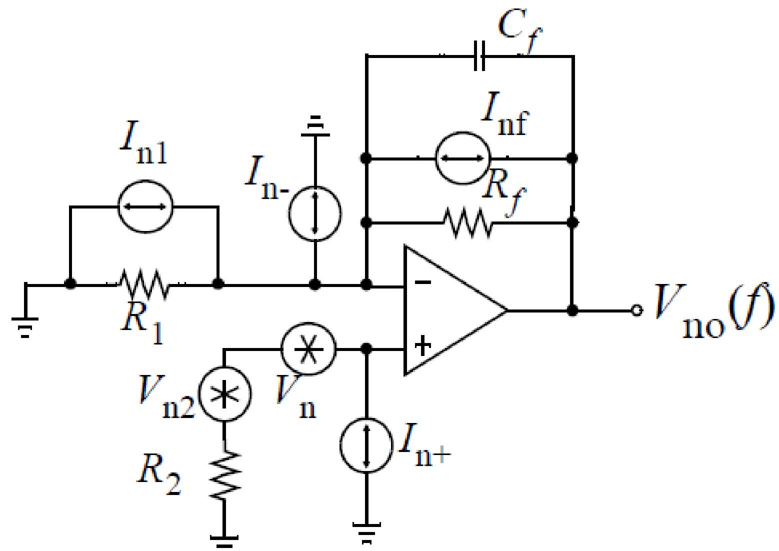
$I_{n+}(f)$ ignored $\Rightarrow V^2_{no} = V^2_n$

Actual $V^2_{no} = V^2_n + (I_{n+}R)^2$

Opamp Example



circuit



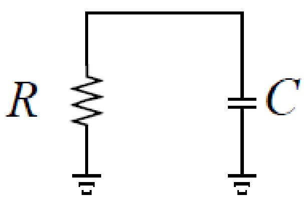
equivalent noise circuit

- Use superposition — noise sources uncorrelated
- Consider I_{n1}, I_{nf} and I_{n-} causing $V_{no1}^2(f)$

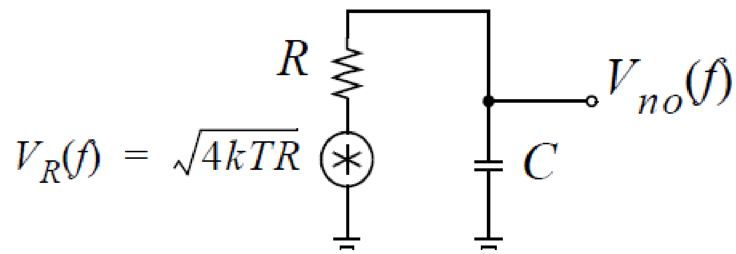
$$V_{no1}^2(f) = (I_{n1}^2(f) + I_{nf}^2(f) + I_{n-}^2(f))\left|\frac{R_f}{1 + j2\pi f R_f C_f}\right|^2 \qquad (41)$$

Capacitors

- Capacitors and inductors do not generate any noise but … they accumulate noise.
- Capacitor noise mean-squared value equals kT/C when connected to an arbitrary resistor value.



$$f_0 = \frac{1}{2\pi RC}$$



- Noise bandwidth equals $(\pi/2)f_o$

$$V^2_{no(rms)} = V_R^2(f)\left(\frac{\pi}{2}\right)f_o = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right)$$

$$V^2_{no(rms)} = \frac{kT}{C} \tag{38}$$

Capacitors Noise Example

- At 300 $°K$, what capacitor size is needed to have 96dB dynamic range with 1 V rms signal levels.
- Noise allowed:

$$V_{n(rms)} = \frac{1V}{10^{96/20}} = 15.8\ \mu V \text{ rms} \tag{39}$$

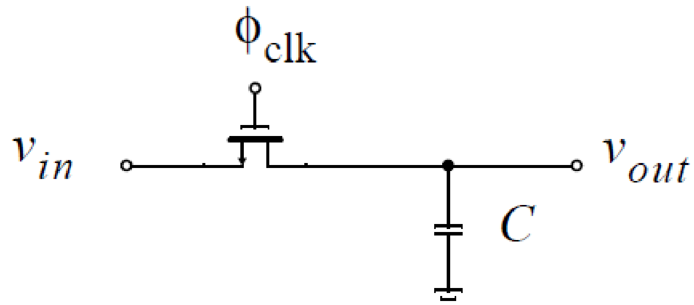
- Therefore

$$C = \frac{kT}{V^2_{n(\text{rms})}} = 16.6pF \tag{40}$$

- This min capacitor size determines max resistance size to achieve a given time-constant.

Sampled Signal Noise

- Consider basic sample-and-hold circuit



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- When ϕ_{clk} goes low, noise as well as signal is held on C.

  — an rms noise voltage of $\sqrt{kT/C}$.
- Does not depend on sampling rate and is independent from sample to sample.
- Can use "oversampling" to reduce effective noise.
- Sample, say 1000 times, and average results.