Chapter 2

Microphones and loudspeakers

2.1 Microphones

2.1.1 Definition

Microphones are electroacoustic transducers which transform acoustic vibrations into electrical signals.

2.1.2 Principles

The displacement of the diaphragm (a membrane or a ribbon) in a sound pressure field results in the modification of an electrical variable of the circuit connected to this diaphragm:

- in carbon microphones (used in old telephone handsets), an electrical resistance was varied;
- in electrostatic microphones, the capacitance of a capacitor having the diaphragm as one of the electrodes is varied;
- in crystal microphones, piezoelectric effects create an electric voltage;
- in electrodynamic microphones, the displacement in a magnetic field of a coil tied to the diaphragm creates a voltage called the electromotive force (EMF).

In each case, the modification of the electrical variable results in an output voltage, which we consider as an audio signal.
2.1.3 Microphones’ properties

The sensitivity is expressed in decibels (dB) and is defined by:

\[ S(\text{dB}) = 20 \log_{10} \left( \frac{e}{p} \right) \]

In this definition, we see the ratio of the RMS output voltage \( e \) (volts) created by the microphone to the RMS acoustic pressure \( p \) (pascals) on the membrane.

Example: A microphone with a sensitivity \( S = -50 \text{ dB} \) outputs a voltage of 10 mV (RMS) for a 104 dB sound pressure level. Indeed:

\[ 104 \text{ dB} = 20 \log_{10} \left( \frac{p}{2 \times 10^{-5}} \right) \Rightarrow p = 3,17 \text{ Pa} \]

\[-50 \text{ dB} = 20 \log_{10} \left( \frac{10^{-2}}{3.17} \right) \]

The frequency response: the sensitivity of the microphone is expressed as a function of the frequency. The (useful) bandwidth of the microphone is the frequency range within which the variations of the sensitivity are limited to (for example) ±3 dB.

The directivity (diagram): is expressed by the variation of the sensitivity as a function of the angle of incidence of the sound wave on the membrane. Three typical directivities are omni-directional, bi-directional and (ultra-)directional. Among directional microphones are the cardioid ones, but also sub-, super- and hyper-cardioid microphones.

Electrical impedance: often expressed at 1kHz, it is the impedance seen by the amplifier connected at the output of the microphone. This impedance can range from 10 Ω to 100 kΩ.

Only electrostatic microphones will be described in this course. Their operation is based on the modification of the capacitance of a condenser whose the first electrode is in a fixed position and the second one (the membrane) is moved by the acoustic vibrations in the sound field.

There are two main types of electrostatic microphones:
• the condenser microphones (dielectric = air) use an external power supply to provide the dc polarizing voltage (see figure 2.1);
• the electret microphones (dielectric = prepolarized material) have an internal permanent polarizing voltage.

2.1.4 Condenser microphones with an external polarizing voltage: basic equations and frequency response

In figure 2.2, $E_0$ is the external dc polarizing voltage applied to the condenser. The output resistance $R_0$ has a very high value ($\simeq 1 \, \text{G} \Omega$).

There exists an electrostatic force of attraction between the condenser’s electrodes, given by:

$$F_e = \frac{e_0 A V^2}{2(d - X)^2}$$

$A$ is the surface of an electrode, $e_0$ is the permittivity of the air and $X$ is the displacement of the diaphragm (measured from its equilibrium position, when there’s no sound and no polarisation voltage $E_0$). $V$ and $d$ are (respectively) the voltage across the electrodes and the equilibrium distance between them.

If we consider the action of $E_0$ alone (static conditions: no sound pressure on the diaphragm), the membrane moves to the position $X = X_0$ which
Figure 2.2: Electrical circuit of the condenser microphone. English translations: "électrode mobile" = moving electrode, "électrode fixe" = motionless electrode.
corresponds to the equality of the electrostatic force of attraction and the elastic force $K_M X_0$ created by the membrane’s stiffness ($K_M$):

$$F_e = K_M X_0$$

Under dynamic conditions (additional action of the sound pressure $p(t)$ on the diaphragm), the displacement of the diaphragm obeys the following differential equation:

$$m_M \frac{d^2 x}{dt^2} + R_M \frac{dx}{dt} + K_M (X_0 + x) = Ap + F_e(V, X)$$

in which $m_M$ is the mass of the membrane, $R_M$ is its mechanical resistance and $K_M$ its stiffness. The displacement $x$ is the (small) increase of $X_0$ due to the pressure on the membrane ($X = X_0 + x$).

In the previous equation, $x$, $p$, $V$ and $X$ depend on time.

The electrostatic force $F_e(V, X)$ can be linearised if we consider that $x$ and $v (= V - E_0)$ are very small quantities, compared to $X_0$ and $E_0$:

$$m_M \frac{d^2 x}{dt^2} + R_M \frac{dx}{dt} + K_M (X_0 + x) = Ap + F_e(E_0, X_0) + K_b v + K_0 x$$

(2.1)

where

$$K_b = \frac{E_0 \varepsilon_0 A}{(d - X_0)^2} = \frac{E_0 C_0}{d - X_0} \quad \text{and} \quad K_0 = \frac{E_0^2 \varepsilon_0 A}{(d - X_0)^3}$$

One must now find a relation between the voltage $v$ and the displacement $x$. Consider that the electric charge appearing on the condenser’s electrodes is expressed by:

$$q = CV = \frac{\varepsilon_0 A (E_0 + v)}{d - X_0 - x} = C_0 E_0 + C_0 v + \frac{C_0 E_0 x}{d - X_0} + \text{second order terms}$$

Therefore, the electric current is given by the first-order (time) derivative of the electric charge $q$:

$$i = C_0 \frac{dv}{dt} + K_b \frac{dx}{dt}$$

The second order terms are again neglected.
Open-circuit sensitivity

One can derive the open-circuit sensitivity of the condenser microphone 
\[ S = 20 \log \left( \frac{v_{\text{rms}}}{P_{\text{rms}}} \right) \] from the previous equations. Without any load impedance, 
the output voltage is \( V = E_0 + v \): see figure 2.2. The d.c. component \( E_0 \) of 
this voltage is blocked by the decoupling capacitor \( C_c \), which gives only \( v \) at 
the output of the microphone circuit. 

With this in mind, we will now re-use the previous equation giving the 
electric current, but we will first express this equation in the frequency do-
main (for sinusoidal signals). Introducing the complex symbols \( \pi \) et \( \nu \), such that:

\[ x(t) = \Re \{ x e^{j\omega t} \} \]

the previous equation gives (since \( i = \frac{v}{R_0} \)):

\[ \frac{\nu}{R_0} = j\omega C_0 \pi + j\omega K_b \pi \]

As the resistance \( R_0 \) has a very high value, the current \( i = \frac{v}{R_0} \) can be 
neglected in the previous equation, except at very low frequencies, which 
gives:

\[ j\omega C_0 \pi + j\omega K_b \pi = 0 \]

We then include this relation into equation (2.1), and we obtain (again 
in the frequency domain):

\[-\omega^2 m_M \pi + j\omega R_M \pi + (K_M - K_0) \pi + \frac{K_b^2}{C_0} \pi = A \pi \]

In the previous equation, we have taken into account the forces’ equilib-
rium under static conditions:

\[ F_c(E_0, X_0) = K_M X_0 \]

An additional simplification arises if the constants \( K_0 \) and \( K_b \) are develop-
ed:

\[-\omega^2 m_M \pi + j\omega R_M \pi + K_M \pi = A \pi \]

Finally, the open-circuit sensitivity of the condenser microphone is the 
following:
Figure 2.3: Typical frequency response of a condenser microphone, for two different values of the mechanical resistance $R_M$ (sensitivity in dB vs frequency).

$$S(\text{dB}) = 20 \log \frac{v_{\text{rms}}}{p_{\text{rms}}} = 20 \log \left| \frac{v}{p} \right|$$

$$= 20 \log \left( \frac{AE_0}{(d - X_0)\sqrt{\left(\omega^2 m_M - \frac{1}{C_M}\right)^2 + \omega^2 R_M}} \right)$$

in which $C_M$ is the compliance (or the inverse of the stiffness $K_M$).

Figure 2.3 illustrates the frequency response corresponding to this equation.

The resonance frequency is given by $f_0 = \frac{1}{2\pi \sqrt{m_M} C_M}$. At this particular frequency, there’s a peak in the frequency response which corresponds to the upper limit of the useful bandwidth. Therefore, there’s a clear interest to push this resonance frequency as far as possible, towards the high frequencies. How can this be obtained? Mainly by using a very light membrane, made of nickel, aluminium or a thin sheet of Mylar covered by a thin metallic layer.

Another approach would be to decrease the diaphragm’s compliance $C_M$, but this would also decrease the sensitivity in the useful bandwith:

$$S_0 = 20 \log \left( \frac{AE_0 C_M}{d - X_0} \right)$$
To increase the sensitivity $S_0$, one can increase the polarizing voltage $E_0$ (typical value: 200 V) or decrease the distance $d$ between the electrodes (typical value: 20 $\mu$m).

The main effect of the mechanical resistance $R_M$ is to damp the resonance peak in the frequency response. By increasing its value, it is therefore possible to enlarge the useful bandwidth, without changing the resonance frequency (see figure 2.3). The mechanical resistance can in fact be increased by the \emph{acoustic resistance} of small holes (resistance to air flow) perforated in the back plate of the capacitor: see figure 2.1.

Beyond the resonance frequency, the sensitivity decreases by 12 dB per octave ($-20\log\omega^2$).

\subsection*{2.1.5 Electret microphones}

Electrets are prepolarized materials having an internal permanent electrostatic charge (ex.: polytetrafluoroethylene).

They can be used in electrostatic microphones to get rid of the external polarizing voltage.

Two possible configurations:

- either the capacitor (fixed) backplate is coated with the electret material;
- or the metallic membrane itself is coated with the electret.

The properties (frequency response, ...) of electret microphones are very similar to those provided by the condenser microphone described before. The \textit{internal} polarizing voltage $E_0$ can be as high as 100 V.

However, an electret microphone needs a small dc power supply (5 to 9 V), often provided by batteries. This is required to operate the pre-amplification of the output voltage.

Due to their small size, electret microphones are well suited for compact applications:

- "Tie-clip" microphones for speech reinforcement in conferences;
- microphones for mobile phones;
- head-worn microphones for singers in a stage performance.

The last ones generally have special directional characteristics (cardioid or hypercardioid, see next section).
2.1.6 Directional microphones

In pressure-gradient microphones, the acoustic pressure is active on both sides of the diaphragm (figure 2.4).

However, the value of the acoustic pressure is different at both sides, because the sound wave must travel the additional equivalent distance $2d$ (the (acoustic) path) between the front and the back sides. Let’s call $p_1$ the pressure on the front and $p_2$ the pressure on the back side.

The difference between $p_1$ and $p_2$ therefore depends more on the phases of the acoustic pressures than on their amplitude.

Consider such a microphone located at point A in a sound field. The angle of incidence of the sound wave on the diaphragm is $\theta$ (fig. 2.5).

The path is maximum if the direction of propagation is perpendicular to the membrane ($\theta = 0$) and minimum if $\theta = 90^\circ$. In the following, we assume that the acoustic path in the case of non-perpendicular incidence is $2d \cos \theta$. If we apply this model to equation (1.1) of the first chapter, then:
Figure 2.5: Bypass of the diaphragm in a pressure gradient microphone in the case of non-perpendicular incidence. English translations: see previous figure.
Figure 2.6: Directivity of the pressure gradient microphone in \( \cos \theta \) (figure-eight microphone)

\[
p_1 = P_M \cos (\omega t - kx_A + kd \cos \theta)
\]

\[
p_2 = P_M \cos (\omega t - kx_A - kd \cos \theta)
\]

where \( x_A \) is the coordinate of point A along the direction of propagation and \( P_M \) is the amplitude of the acoustic pressure which is assumed unchanged when passing from the front to the back side of the diaphragm.

The resulting force on the diaphragm is proportional to the difference between the acoustic pressures applied to both sides, and therefore to \( \sin (kd \cos \theta) \).

- if \( kd \) is small, \( \sin (kd \cos \theta) \simeq kd \cos \theta \), and the directivity of the microphone follows a spatial distribution in \( \cos \theta \) at all frequencies, which gives two lobes of directivity shown in fig. 2.6.

- if \( kd \) is greater than (about) 0.3, the directivity depends on frequency (through \( k \)). Additional side lobes are created in the directivity diagram and their number increases with frequency.
Pressure gradient microphones are usually designed to operate in their bi-directional mode ($kd < 0.3$); they are often called *figure-eight* microphones, with reference to the particular shape of their directivity diagram.

An example of application of the pressure gradient microphone is the recording of an interview in a TV or a radio studio, when both people are facing each other and the microphones’ lobes are oriented towards the interviewer and the interviewee, respectively. Possible noise sources generated by technical equipments are therefore not recorded if they are situated along the direction $\theta = 90^\circ/270^\circ$. 
**Directional microphones**

Contrary to bidirectional microphones, directional microphones have a (unique) main lobe of sensitivity.

A *directional* microphone can be obtained by a pressure and a pressure gradient microphones connected in series. Except at very high frequencies, the pressure microphone is omnidirectional (constant directivity); therefore, the output voltage produced by both microphones connected in series is:

\[
e(\theta) = e_p + e_{\Delta p} \cos \theta = e_p (1 + \alpha \cos \theta)
\]

where \(e_p\) is the output voltage of the pressure microphone and \(e_{\Delta p}\) is the output voltage of the pressure gradient microphone, under perpendicular incidence. The factor \(\alpha\) is the ratio of the sensitivities of both microphones under perpendicular incidence.

By modifying \(\alpha\), one can obtain different directivity diagrams (figure 2.7). In particular, a **cardioid microphone** is obtained with \(\alpha = 1\).

It is of course possible to realize directional microphones with a much narrow (selective) sensitivity lobe than cardioid microphones. These can be called ultra-directional microphones: they will not be studied in this course.

### 2.2 Loudspeakers

#### 2.2.1 Definition

Loudspeakers are electroacoustic transducers that transform (electrical) audio signals into acoustic waves.

#### 2.2.2 Principles of operation

The electrical audio signal is amplified at the necessary energy level to control the displacement of a membrane (*diaphragm*), which itself induces the displacement of the neighbouring air mass and creates acoustic vibrations.

Ideally, the acoustic pressure wave created by the membrane should be a perfect reproduction of the audio signal (*high fidelity*). In practice however, differences can be observed between both time evolutions, particularly for the *electrodynamic* loudspeakers which are used in most applications.

In this course, only the loudspeakers in which the diaphragm is directly acting on the air mass are studied. Some loudspeakers use *horns* to provide a better coupling between the diaphragm and the air load, but they will not be described here.
Figure 2.7: Some directivity diagrams which can be realized by varying the factor $\alpha$ in directional microphones. From left to right and top down: subcardioid, cardioid, super- and hyper-cardioid.
2.2.3 Loudspeakers’ properties

**Frequency response**: sound pressure level (SPL) in free field as a function of the frequency of the (constant amplitude) input audio signal (figure 2.8). The SPL is generally measured at 1 m distance from the membrane, in the axial direction (perpendicular to the diaphragm, passing through its center).

**Directivity coefficient**: ratio of the rms pressure measured in a specific direction to the rms pressure measured at the same distance in the axial direction. This ratio generally depends on frequency \( f \). Symbol: \( Q_p(f, \alpha) \), where \( \alpha \) is the angle between the specific direction and the axial direction. The directivity diagram (figure 2.9) gives the value of \( 20 \log_{10} (Q_p(f, \alpha)) \) versus the angle \( \alpha \) (with \( f \) as a parameter).

**Electrical impedance**: average value of the input electrical impedance, measured in the useful bandwidth and specified by the manufacturer. For electrodynamic loudspeakers, it is somewhat higher than the d.c. input resistance.

**Electrical rated power** \( P_n \): maximum input power (watts) specified by
Efficiency: ratio of the radiated acoustic power to the electrical input power, expressed in percents.

2.2.4 Electrodynamic loudspeakers

This is the most common type of loudspeakers for hi-fi applications. They are most often used by groups of two or three items in an enclosure in order to cover the widest possible frequency interval.

It is indeed difficult to cover the whole audio bandwidth with only one loudspeaker, as it will be seen later.

Figure 2.10 shows a sectional view of the electrodynamic loudspeaker. The following elements can be seen on this figure:

1. permanent magnet.
2. moving coil traversed by the electric current (the amplified audio signal).
3. the spider: an elastic suspension supporting the coil and connecting it to the rigid chassis or basket. It only allows for the axial movement of the coil (from left to right on figure 2.10).
Figure 2.10: Sectional view of the electrodynamic loudspeaker (see text for the description of the numbered items).
4. the membrane (diaphragm or cone), which is connected to the coil. The cone has a light weight (in paper or aluminium) in order to attenuate inertial effects and the aperture of the cone is chosen to increase its stiffness (aperture of 100 to 130 degrees).

5. the metallic chassis or basket.

6. the surround elastic suspension.

2.2.4.1 Basic equations

Figure 2.11 shows the electrical model of the loudspeaker: $R_E$ and $L_E$ are the moving coil resistance (about 5 Ω) and inductance, respectively. The induced voltage $e_{ind}$ created by the coil’s movement in the magnetic field is given by:

$$e = B l \frac{dx}{dt}$$

where $B$ is the magnetic induction, $l$ is the length of the coil’s wire and $\dot{x} = \frac{dx}{dt}$ is the velocity of the coil in the gap of the permanent magnet.
Moreover, the combined action of the coil's current \(i(t)\) and the magnetic induction \(B\) creates the force \(F = Bl\) that moves the coil and the diaphragm in a direction perpendicular to the radial magnetic field.

The following equations only describe the loudspeaker’s operation at low frequencies, for which it can be assumed that the cone acts like a circular piston of mass \(m_M\) (membrane + coil + suspensions) and radius \(a\).

At medium and high frequencies, the vibration modes of the membrane are more complicated and the simple model of the piston is not accurate enough.

We further assume that the piston is placed in an infinite baffle, i.e. a rigid plate with a hole in its center having the same circular shape and radius \(a\) than the piston. Its role is to impede the acoustic wave emitted by the cone to the rear direction to interfere with the wave emitted to the front. As these waves are emitted "out-of-phase" (a compression on one side of the diaphragm corresponds to a decompression on the opposite side), radiations can be eliminated at low frequencies if no baffle is present.

Enclosing the loudspeaker in a box is another (more practical) way of impeding these interferences between front and rear emitted waves.

In the case of an infinite baffle, the air load in contact with the cone can be modelled by a mechanical impedance:

\[
Z_{\text{air}} = \frac{\pi \rho_0 a^4 \omega^2}{c} + j \frac{16 \rho_0 a^3 \omega}{3} \quad k a \ll 1
\]

Note: \(\rho_0 = 1.2 \text{ kg/m}^3\) is the air density and \(c\) is the speed of sound.

A mechanical impedance is in fact used in a mechanical model to represent a reaction force, opposed to the movement and proportional to the velocity. In this case, the air load opposes to the coil’s movement a force given by \(F = Z_{\text{air}} \dot{x}\).

If a circular piston is placed in an infinite baffle, it can be shown that the acoustic pressure produced in the axial direction at the distance of \(r\) meters can be expressed by (if \(r \gg a\)):

\[
p(r, t) = \Re \left( j \frac{\omega \rho_0 a^2}{2r} \hat{x} e^{j(\omega t - kr)} \right) \quad r \gg a
\]

Let’s now consider the basic equations. The movement of the cone obeys the following equation (expressed in the frequency domain):
\[ j\omega \dot{x} \left( m_M + \frac{16\rho_0 a^3}{3} \right) + \left( R_M + \frac{\pi \rho_0 a^4 \omega^2}{c} \right) \dot{x} + \frac{\dot{x}}{j\omega C_M} = Bl_i \]

\[ Bl_i = Bl \frac{e_g - e_{\text{ind}}}{Z_{es}} = Bl \frac{e_g}{Z_{es}} - (Bl)^2 \frac{\dot{x}}{Z_{es}} \]

where \( R_M \) and \( C_M \) are the mechanical resistance and compliance, respectively, whereas \( Z_{es} \) is the static electrical impedance of the coil \((Z_g + R_E + j\omega L_E)\). The compliance \( C_M \) models the elastic effect of the cone and its suspensions.

Note that the air load has been modelled by an inductance term (equivalent mass) and a resistance term, in the mechanical equation.

Some simplifications to these basic equations can be operated. For example at low frequencies, the static electrical impedance can be approximated by its real part, the resistance \( R_{es} \). Therefore, if the first term of the previous equation is simply written \( Z_{MT} \dot{x} \), then the acoustic pressure at 1m distance becomes:

\[ p_{\text{rms}} = \frac{\omega \rho_0 a^2}{2} \frac{Bl (e_{g,rms})}{R_{es} |(Bl)^2 + Z_{MT}|} \quad a \ll 1 \text{ m} \]

The frequency response in free field is (by definition) the acoustic pressure produced at 1 m distance in the axial direction vs frequency, for a constant value of the input voltage \( e_{g,rms} \). The previous equation therefore clearly shows the existence of a resonance frequency \( f_0 \) which corresponds to the cancellation of the imaginary part of \( Z_{MT} \):

\[ f_0 = \frac{1}{2\pi \sqrt{\left( m_M + \frac{16\rho_0 a^3}{3} \right) C_M}} \]

At frequencies below the resonance, the cone’s movement is predominantly ruled by the compliance. The value of the acoustic pressure is therefore proportional to the square of the frequency \((\omega^2)\), i.e. an increase of 12 dB per octave.

At greater frequencies (beyond the resonance), the equivalent mass is predominant and would impose a constant response as a function of frequency. However, one must recall that the impedance of the air load has a real part which is proportional to \( \omega^2 \): this would rather impose a 6 dB per octave
decrease at very high frequencies. The theoretical frequency response of the loudspeaker therefore looks like the curve shown in figure 2.12.

Furthermore, one must also recall that the model previously described is a low frequency model. At high frequencies, as already mentioned, the movement of the membrane becomes more intricate and the coil’s inductance $L_E$ is no longer negligible compared to the resistance $R_{es}$. The complete (broadband) theoretical frequency response is given without demonstration in figure 2.13.

### 2.2.4.2 Improvement of the frequency response

at low frequencies: the extension of the useful bandwidth towards low frequencies requires the decrease of the resonance frequency $f_0$. This is obtained by increasing the diameter of the cone, since the total
mass and the equivalent mass of the air load are increased. Another beneficial effect is that the peak at the resonance frequency is damped by the increase of the air load’s equivalent resistance, which contains a factor proportional to \(a^4\).

A second method would be to increase the compliance \(C_M\) (by releasing the stiffness of the suspensions), but this would also result in the amplification of the coil’s displacement which could partially leave the magnet’s gap and create distortions in the acoustic signal (in fact, the force \(Bli\) would now depend on the position of the coil in the gap, introducing non-linearities in the equations). Also, great displacements \(x\) of the cone could create elastic forces which are no longer proportional to \(x\).

Let’s finally note that the enclosure (box) that usually contains the loudspeaker (rather than an infinite baffle) can also be designed to improve the low frequency response, below the resonance.

**at high frequencies**: the method to extend the frequency response towards high frequencies is complementary to the previous one, i.e. increasing the resonance frequency. This can be obtained by decreasing the diameter of the membrane and/or increasing the stiffness.

The following question therefore arises: how is it possible to obtain a high fidelity loudspeaker that practically will have a broad bandwidth extending to the very low, as well as to the very high frequencies?

Usually, this is obtained by the combination of two or three loudspeakers in the same enclosure, each optimized for a particular frequency range:

- **a woofer**: with a large diameter (typically 20 to 30 cm), covering the low frequency range,

- **a tweeter**: with the smallest size (less than 10 cm), covering the high frequency range,

- **a squawker**: sometimes covering the medium frequencies.

In this loudspeakers’ system, a special filters’ network called the crossover network is applied to the output of the power amplifier. It is designed to supply each loudspeaker of the enclosure with the most appropriate frequency components of the audio signal.
2.2.4.3 Some other properties of the electrodynamic loudspeakers

The efficiency (see definition) of these loudspeakers is relatively weak: no more than some percents.

Their directivity becomes more selective as the product $\omega a$ increases. For a given loudspeaker, the emission is quite omnidirectional at low frequencies and it becomes more selective as the frequency increases (fig. 2.9).

The electrical impedance of the electrodynamic loudspeaker depends on frequency. Its d.c. value (at $f = 0$ Hz) equals $R_E$. As the frequency increases, its value dramatically increases to reach a maximum at the resonance frequency. Its value then rapidly decreases in the useful bandwidth until it reaches a relatively constant value ($Z_{\text{min}}$) which is somewhat greater than the d.c. value $R_E$ (cf. exercise 2 and figure 2.16).

Finally, electrodynamic loudspeakers produce significant distortions in the emitted acoustic signal. Their origin is to be found in:

- the non-linearities of the relation between the driving force, the magnetic field and the audio current,
- the non-linear behaviour of the suspensions’ elasticity.

The amplitude of the distortions generally increases with the power of the input signal.
2.3 Exercises

2.3.1 Exercise 1

A condenser microphone must have the following properties:

- a sensitivity of about -50dB (re. 1V/Pa) in its useful bandwidth;
- a bandwidth extending theoretically up to 20 kHz;
- the size of its diaphragm smaller than the wavelength, in the whole useful frequency interval;
- an external polarizing voltage of 200V;
- a distance between electrodes of 20µm.

The exercise consists in finding the diaphragm’s parameters (surface, mass, thickness, stiffness) such that the previous properties are reached. The diaphragm is built in aluminium (density 2700kg/m³).

It must also be checked that the amplitude of the diaphragm’s displacements is less than the distance between electrodes, in the whole frequency band and for sound pressure levels up to 120 dB.
2.3.2 Exercise 2

A loudspeaker is supplied with an input signal of 10 Vrms, whose frequencies belong to the useful bandwidth of the loudspeaker. The electrical power consumption is 25 watts.

Answer the following questions:

• what is the input impedance?

• what is the efficiency of the loudspeaker and the acoustic power radiated to both sides of the cone?

• what is the cone’s displacement at the frequency of 500 Hz (RMS value)?

• what is the sound pressure level (dB) produced in the useful bandwidth at 1m and 4m from the loudspeaker’s diaphragm (axial direction)?

Data:

• resonance frequency: 112 Hz;

• diameter of the cone: 16cm;

• moving mass (including the air load): 0.022 kg;

• $B_l = 10$ teslas.m;

• input impedance measured at the resonance frequency = $29\Omega$.

2.3.3 Exercise 3

In a theatre, two identical electrodynamic loudspeakers are placed on the stage, as indicated on the figure 2.15. Both are mounted in an infinite baffle. One is supplied with an input signal whose central frequency is 200 Hz ($1V_{rms}$) and the other with an input signal whose central frequency is 300 Hz ($1V_{rms}$ also). Both signals are considered as not correlated.

The loudspeakers’ directivity coefficient in the interesting frequency range is the following: $Q_\beta = \cos\beta$ ($\beta$ is the angle between the direction of interest and the axial direction).

The sound engineer places two figure-eight microphones at the same position situated 3m away from the loudspeakers. These microphones are identical and connected in series, one is oriented such that the direction of
Figure 2.15: Data for exercise 3. English translation: "Source perturbatrice"
= noise source, disturbance.
maximum sensitivity points to the left loudspeaker. The other microphone is oriented such as to attenuate (and possibly cancel) in the recorded signal the influence of a noise source situated as indicated on the figure 2.15.

What are:

• the (mechanical) resonance frequency of the loudspeakers;

• the sound pressure level produced by each loudspeaker at the microphones’ position;

• the orientation of the second microphone such as to cancel the contribution of the noise source in the recorded signal;

• the rms value of the voltage produced by the two microphones connected in series.

Data:

• $c = 344 \text{ m/s}$

• $\rho_0 = 1.2 \text{ kg/m}^3$

• diameter of the loudspeakers’ cone: 16cm;

• moving mass (for one loudspeaker): 0.022kg;

• mechanical compliance of a loudspeaker: $8 \times 10^{-5} \text{ m/N}$;

• mechanical resistance of a loudspeaker: $4 \text{ N.s/m}$;

• electrical resistance and inductance of a loudspeaker’s coil: $8 \Omega$ and $2\text{ mH}$;

• magnetic induction times length of the coil’s wire: $Bl = 10 \text{T.m}$;

• microphones’ axial sensitivity: $-70 \text{ dB}$, re. $1\text{ V/Pa}$.
2.3.4 Exercises’ solutions

2.3.4.1 Exercise 1

- Surface of the diaphragm: $2.27cm^2$ (diameter = minimum wavelength’s value: 17mm);
- Stiffness = 718000N/m;
- Mass = 0.0455g (for a resonance frequency at 20 kHz);
- Diaphragm’s thickness: 74µm.

The displacements’ amplitude in a sound field of 120 dB is approximately 0.0063µm, and therefore significantly less than the distance between electrodes.

2.3.4.2 Exercise 2

Figure 2.11 shows that the input impedance of the loudspeaker is (neglecting the output impedance of the amplifier):

$$Z_{in} = \frac{e_i}{i} = R_E + j\omega L_E + \frac{Bl}{i} \frac{dx}{dt} \quad (2.2)$$

Moreover, if $Z_{MT}$ is the total mechanical impedance, considering that $Bli = Z_{MT} \frac{dx}{dt}$, the input impedance is written as the following:

$$Z_{in} = R_E + j\omega L_E + \frac{(Bl)^2}{Z_{MT}} \quad (2.3)$$

At low frequencies, the term $j\omega L_E$ can be neglected. Therefore, figure 2.16 shows the evolution of the input impedance vs frequency.

At $f = 0Hz$, the input impedance is real and equal to $R_E$.

At the resonance frequency ($f_0 = 112Hz$), it is also real and reaches its maximum value 29Ω.

In the useful bandwidth (beyond $f = 112Hz$), the electrical dissipated power $W_{in}$ is:

$$W_{in} = \Re \left( \frac{1}{Z_{in}} \right) e_{g,\text{rms}}^2 \quad (2.4)$$

According to figure 2.16, the imaginary part of $Z_{in}$ is much lower than the real part in the useful bandwidth. We can therefore approximate the real part of $\frac{1}{Z_{in}}$ by $\frac{1}{R_E}$. The previous equation therefore leads to $R_E = 4\Omega$. 

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Figure 2.16: Typical evolution of the input impedance (Ω) of the electro-dynamic loudspeaker vs frequency (real and imaginary parts), according to the low frequency electroacoustic model English translation: "Partie réelle, imaginaire" = real, imaginary part.
Figure 2.16 has been plotted with $R_{MT}$ (the real part of $Z_{MT}$) equal to 4 N.s/m, such as to obtain at the resonance frequency:

$$Z_{in} = R_E + \frac{(Bl)^2}{R_{MT}} = 29\Omega$$

(2.5)

The mechanical compliance is given by:

$$C_M = \frac{1}{m_M * (2\pi f_0)^2} = 9.18 \times 10^{-5} \text{m/N}$$

(2.6)

The air load applies to the diaphragm’s movement a force given by: $F = Z_{air} \frac{dx}{dt}$. Therefore, the acoustic power radiated in the propagation medium is:

$$W_{rad} = \Re(Z_{air}) \left(\frac{dx}{dt}\right)^2_{rms} = \frac{\pi \rho_0 a^4 \omega^2}{2c} \left(\frac{dx}{dt}\right)^2_{rms}$$

(2.7)

In the useful frequency range, the rms value of the diaphragm’s velocity is determined by the total mass, including air load (cf. theory):

$$\left(\frac{dx}{dt}\right)_{rms} \simeq \frac{Bl \ \epsilon_g \text{rms}}{R_E |j\omega M|} = \frac{1140}{\omega}$$

(2.8)

Applying both previous equations leads to a radiated power $W_{rad} = 0.59W$ and a loudspeaker’s efficiency of 2.4 percent (since: $W_{in} = 25W$).

The rms value of the diaphragm’s displacement at 500 Hz (in the useful bandwidth) is directly given by equation 2.8 if we divide by $\omega$, i.e. 0.12mm.

Finally, the sound pressure level produced at $r$ meters is given (in the useful bandwidth) by eq. 2.8 multiplied by $(\frac{\omega \rho_0 a^2}{2r})$ (cf. theory). This leads to 4.4Pa at 1m (rms pressure, or 107 dB) and 1.1Pa (95dB) at 4m from the loudspeaker’s cone.

### 2.3.4.3 Exercise 3

The resonance frequency:

$$f_0 = \frac{1}{2\pi \sqrt{\left(\frac{m_M}{C_M} + \frac{16\rho_0 a^3}{3}\right)}} = 112Hz$$

(2.9)

Frequencies 200Hz and 300Hz are therefore in the useful bandwidth.

The sound pressure level at $r$ meters from the loudspeaker is (cf. theory):
\[ p_{\text{rms}} = \frac{\omega p_0 a^2 B l e_{g,\text{rms}}}{2r R_E |Z_{MT}|} \]  
\hspace{1cm} (2.10)

If we assume that the mass term is predominant in the total mechanical impedance \( Z_{MT} \) (which is the case beyond the resonance frequency), the angular frequency \( \omega \) is cancelled in the numerator and the denominator of 2.10, which gives 0.063\( \text{Pa} \) or 70dB at 3m, in the axial direction of the loudspeaker.

This pressure level must be corrected by the directivity coefficient of the loudspeaker, i.e. 0.063 \( \cos(56.4^\circ) \)=0.035\( \text{Pa} \) or 64.8dB.

Both loudspeakers together produce a rms acoustic pressure of 
\[ \sqrt{p_{\text{rms},1}^2 + p_{\text{rms},2}^2} = 0.049\text{Pa} \text{ or 67.9dB (signals not correlated)}. \]

The acoustic pressure created by the noise source at the microphones' location is now called \( p(t) \). The voltage at the output of the connected microphones is therefore:

\[ e(t) = \sigma_0 (\cos\theta_1 + \cos\theta_2) \]  
\hspace{1cm} (2.11)

• \( \sigma_0 \) (V/\( \text{Pa} \)) is the axial sensitivity of each microphone. Its value is: 3.16 \( 10^{-4} \) V/\( \text{Pa} \) (-70dB re. 1V/\( \text{Pa} \)),

• \( \theta_1 \) is the angle between the direction of the noise source and the axial (reference) direction of microphone 1 (-78.6\( ^\circ \)),

• \( \theta_2 \) is the same angle, but for microphone 2 (\( \theta_1 - \alpha \)).

Note: Positive angles are counted clockwise.

The sum of both cosine functions in 2.11 must be zero, if the noise source’s contribution is to be cancelled, which gives \( \alpha = 22.8^\circ \).

Finally, the expression of the output voltage (microphone 1) due to both loudspeakers (LS1 and LS2) is the following:

\[ e_1(t) = \sigma_0 \cos0^\circ p_{LS1}(t) + \sigma_0 \cos112.8^\circ p_{LS2}(t) \]  
\hspace{1cm} (2.12)

The same holds for microphone 2:

\[ e_2(t) = \sigma_0 \cos22.8^\circ p_{LS1}(t) + \sigma_0 \cos90^\circ p_{LS2}(t) \]  
\hspace{1cm} (2.13)
The total output voltage $e_1(t) + e_2(t)$ includes the contributions of signals emitted by LS1 (frequency range around 200 Hz) and signals emitted by LS2 (around 300 Hz). Since both contributions are not correlated, the total rms output voltage is:

$$e_{\text{rms,total}} = \sigma_0 \sqrt{((1 + \cos 22.8^\circ)p_{\text{rms,1}})^2 + ((\cos 112.8^\circ)p_{\text{rms,2}})^2} = 21.7 \mu V$$

(2.14)