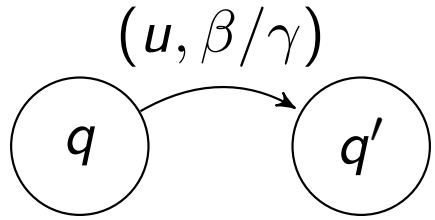
Introduction to computability
Tutorial 6

Pushdown Automata and Context Free Languages

06 November 2018

How to draw pushdown automata

A transition $((q, u, \beta), (q', \gamma))$ is represented by



1. Describe the language accepted by the pushdown automata $M = (Q, \Sigma, \Gamma, \Delta, Z, s, F)$ where:

- $Q = \{s, p, q, f\}$;
- $\Sigma = \{a, b, c\}$;
- $\Gamma = \{A, Z\}$;
- $F = \{f\}$;
- Δ contains the following transitions:
  - $(s, a, \varepsilon) \to (s, A)$,
  - $(s, b, \varepsilon) \to (q, \varepsilon)$,
  - $(s, c, A) \to (p, \varepsilon)$,
  - $(q, b, \varepsilon) \to (q, \varepsilon)$,
  - $(q, c, A) \to (p, \varepsilon)$,
  - $(p, c, A) \to (p, \varepsilon)$,
  - $(p, \varepsilon, Z) \to (f, \varepsilon)$.

2. Give a pushdown automata accepting each of the following languages:

- The language generated by the grammar

$$S \to aSa$$
$$S \to bSb$$
$$S \to \varepsilon;$$

- $L = \{a^n b^{2m+n} c^m | n, m \geq 0\}$;
- The language of the words on the alphabet $\{a, b\}$ that contain as many a's as b's;
- $L = \{a^n b^m | 0 < n \leq m \leq 2n\}$.

3. Let M_1 and M_2 be two pushdown automata that accept the context free languages L_1 respectively L_2. Give a pushdown automata that accepts

a) $L_1 \cup L_2$;

b) $L_1.L_2$;

c) L_1^*.

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4. Take $L = \{a^n b^m a^m b^n | n, m > 0\}$

a) Show that this language is context-free.

b) Is it a deterministic context-free language? Explain.

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5. If L is a context-free language and R a regular language, are the following languages context-free?

a) $L \setminus R$

b) $R \setminus L$

Bonus Exercise 6

Let L be the language defined by

$$L = \{a^n b^m \mid n, m \in \mathbb{N} \text{ and } |n - m| = 2\}$$

where $|n - m|$ denotes the absolute value of $n - m$.

1. Give a push-down automata that accepts L.
2. Give a context-free grammar that generates L.
3. If n and m are bounded, what can you say about L?