Introduction to computability
Tutorial 3

Finite Automata

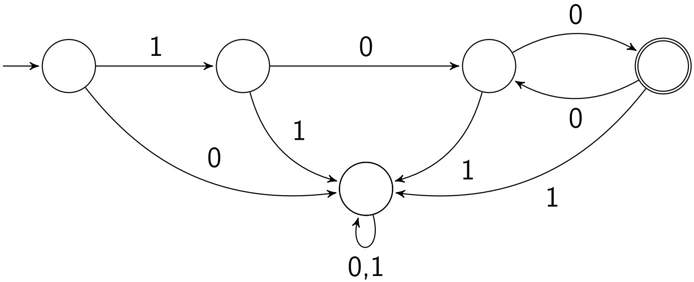
02 October 2018

Finite Automata

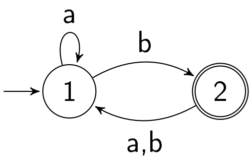
1. Let $L \subseteq \Sigma^*$. Show that if L is a regular language, than so is $\overline{L}$, the the complement of L.
Formally:

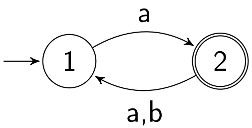
$$\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$$

2. Give a regular expression of the language accepted by this automaton. If the accepted words are considered as being binary representations of integers, determine the arithmetic properties of the accepted words.



3. Consider the two finite automata represented below, that accept respectively the languages L_1 and L_2. Give finite automata that accept the languages $L_1 \cup L_2$, $L_1.L_2$ and L_1^*.





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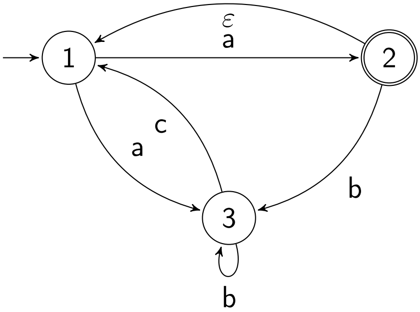
5. Give a regular expression of the language accepted by the following automaton by using the $R(i, j, k)$ - method.



From automata to regular languages

Let M be an automaton and $Q = \{q_1, q_2, \dots, q_n\}$ its set of states. We will denote by $R(i, j, k)$ the set of words that can lead from the state q_i to the state q_j, going only through states in $\{q_1, \dots, q_{k-1}\}$.

$$R(i,j,1) = \begin{cases} \{w \mid (q_i, w, q_j) \in \Delta\} & \text{if } i \neq j \\ \{\varepsilon\} \cup \{w \mid (q_i, w, q_j) \in \Delta\} & \text{if } i = j \end{cases}$$

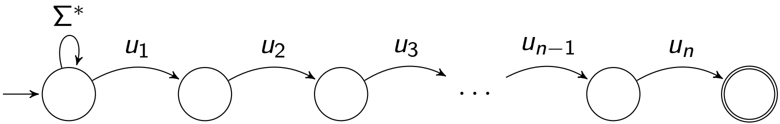
$$\begin{aligned} R(i,j,k+1) \quad = \quad & R(i,j,k) \quad \cup \\ & R(i,k,k) R(k,k,k)^* R(k,j,k) \end{aligned}$$

If q_1 is the initial state,

$$L(M) = \bigcup_{q_j \in F} R(1, j, n+1)$$

Pattern search

We want to look in a text on the alphabet Σ for the word u. We thus want an automaton that accepts $L(\Sigma^* u)$.



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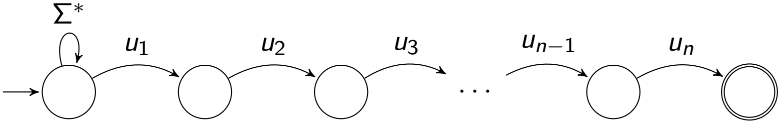

6. Give a deterministic finite automaton that recognizes the pattern "nano".

Grammars

- A grammar is a 4-tuple $G = (V, \Sigma, R, S)$, where
  - V is an alphabet,
  - $\Sigma \subseteq V$ is the set of *terminal symbols* ($V \setminus \Sigma$ is the set of *nonterminal symbols*),
  - $R \subseteq (V^+ \times V^*)$ is a finite set of *production rules*,
  - $S \in V \setminus \Sigma$ is the *start symbol*.
- The language generated by a grammar G is the set

$$L(G) = \{v \in \Sigma^* \mid S \Rightarrow_G^* v\}.$$

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c) the language of the palindromes on $\Sigma = \{a, b\}$, i.e. the language containing the words $w = w_0 w_1 \dots w_n$ such that for all i, $0 \leq i \leq n$ we have that $w_i = w_{n-i}$;

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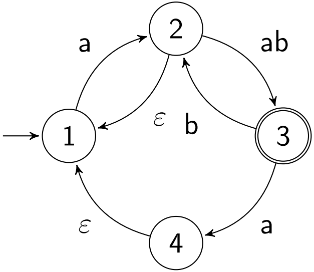
d) the language accepted by the following automaton:



8. Describe the languages generated by the following grammars:

a)	$S \to aSa$	b)	$S \to aS$	c)	$S \to LaR$
	$S \to bSb$		$S \to bS$		$L \to LD$
	$G \to \varepsilon$		$S \to \varepsilon$		$Da \to aaD$
					$DR \to R$
					$L \to \varepsilon$
					$R \to \varepsilon$

Bonus Exercise 3

Give a DFA that accepts the language L containing all the words on the alphabet $\{a, b\}$ that contain neither aab nor bba. In other words,

$$L = \{w \mid w \in \{a, b\}^*, \, aab \notin Fact(w) \text{ and } bba \notin Fact(w)\}$$