Introduction to computability
Tutorial 1

Regular Expressions and Denumerable Sets

18 September 2018

Regular expressions

1. Let R and S be the following regular expressions:

$$R = a^* \cup b^*$$

$$S = ab^* \cup ba^* \cup b^*a \cup (a^*b^*)^*$$

(a) Find a word that belongs to $L(S)$ but not to $L(R)$.
(b) Find a word that belongs to both $L(R)$ and $L(S)$.
(c) Find a word that belongs to $L(R)$ but not to $L(S)$.
(d) Find a word that belongs neither to $L(R)$ nor to $L(S)$.

2. Determine whether the following statements are correct or not on the alphabet $\Sigma = \{a, b\}$:
(a) $aab \in L(b^*a^*b^*a^*)$;
(b) $L(\emptyset^*) = L(\epsilon)$;
(c) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$;
(d) $(L_1.L_2)^* = L_1^*.L_2^*$.

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3. Give a regular expression of the following languages (defined on $\Sigma = \{a, b\}$):
(a) the language whose words contain an odd number of a's;
(b) the language whose words contain exactly once the factor aaa;
(c) the language whose words do not end with b;
(d) the language whose words contain an even number of symbols.

Denumerable sets

4. Are the following sets denumerable?

(a) The set $\mathbb{Z}$ containing all integers.

(b) The set $\mathbb{N} \times \mathbb{N}$ of all pairs.

(c) The set $2^{\mathbb{N}}$ of all subsets of $\mathbb{N}$.

(d) The set $\mathbb{Q}$ containing all rational numbers.

(e) The set of all polynomials with integer coefficients.

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5. Show that
(a) every infinite subset of a denumerable set is itself denumerable;
(b) the difference between a non-denumerable and a denumerable set is not denumerable.

Bonus Exercise 1

Is the set of well formed arithmetic expressions denumerable?
For example $3 * (2 + 4)$ is well formed and $3 + *5$ is not.