Introduction to the Theory of Computation

Final exam

15 January 2016

Closed-book. Duration: 3h30.
Please answer each question on a separate sheet with your name and section. ***Motivate all your answers and give sufficient details.***

1. a) Is the set of all closed intervals of $\mathbb{R}$ with rational bounds denumerable?

   b) Is the set of all regular expressions over a finite alphabet Σ denumerable? What happens when Σ is infinite (but denumerable)?

2. a) Give a DFA that accepts the language

   $$L_1 = \{w \mid w \in \{a,b\}^*,\ N_a(w) = 2 \mod 4\}$$

   where $N_\sigma(w)$ is the number of letters σ contained in the word w.

   b) Give a DFA that accepts the language

   $$L_2 = \{w \mid w \in \{a,b\}^*,\ aa \notin Fact(w)\}$$

   c) Give a regular grammar that generates $\overline{L_1 \cup L_2}$.

3. a) Is the language $\{a^m b^n c^{\max(m,n)} \mid m,n \in \mathbb{N}\}$ regular?

   b) Is the language of all well-parenthesized expressions regular? *Example:* $(()())$.

4. a) State and prove the pumping lemma for context-free languages.

   b) Is the language $L = \{a^i b^j c^k d^l \mid i + l \geqslant j + k\}$ context-free?

5. a) State the Turing-Church Thesis. What type of justification can be given for this thesis?

   b) Are two tape Turing Machines more expressive than the standard definition of a Turing Machine? Explain.

6. a) Do there exist computable functions that are not primitive recursive?

   b) Show that IntegerSqrt$(n) = \lfloor\sqrt{n}\rfloor$ is primitive recursive.

   c) Is IntegerSqrt μ-recursive? Why are μ-recursive functions of interest in computability theory?

7. a) Let M be a Turing Machine. Show that the problem that consists in determining whether M stops on all words of even length is undecidable.
   Hint: Use the empty-word halting-problem.

   b) Why are the languages accepted by a Turing Machine also called "recursively enumerable"? Prove your statement.

8. a) Show that HC $\propto$ TS.

   b) Define the complexity classes P, NP and NPC. What inclusion relations between these classes are known, plausible? Give an example of a problem belonging to each of these classes.