Probability and Statistics

Kristel Van Steen, PhD[2]

Montefiore Institute - Systems and Modeling

GIGA - Bioinformatics

ULg

kristel.vansteen@ulg.ac.be

CHAPTER 4: IT IS ALL ABOUT DATA

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1 An introduction to statistics

1.1 Different flavors of statistics

Probability versus statistics - recapitulation

- Key points about probability

1. Rules → data: Given the rules, describe the likelihoods of various events occurring.
2. Probability is about prediction — looking forward.
3. Probability is mathematics.

- Key points about statistics

1. Rules ← data: Given only the data, try to guess what the rules were. That is, some probability model controlled what data came out, and the best we can do is guess — or approximate — what that model was. We might guess wrong; we might refine our guess as we get more data.
2. Statistics is about looking backward.
3. Statistics is an art. It uses mathematical methods, but it is more than maths.
4. Once we make our best *statistical guess* about what the probability model is (what the rules are), based on looking *backward*, we can then use that *probability* model to predict the *future* →
   The purpose of statistics is to make inference about unknown quantities from samples of data

(DeCaro, S. A. (2003). A student's guide to the conceptual side of inferential statistics from http://psychology.sdecnet.com/stathelp.htm.)

Descriptive statistics

- With **descriptive statistics** we condense a set of known numbers into a few simple values (either numerically or graphically) to simplify an understanding of those data that are available to us.
  - This is analogous to writing up a summary of a lengthy book. The book summary is a tool for conveying the gist of a story to others.
  - The mean and standard deviation of a set of numbers is a tool for conveying the gist of the individual numbers (without having to specify each and every one).

Inferential statistics

- **Inferential statistics** is used to make claims about the populations that give rise to the data we collect.
    - This requires that we go beyond the data available to us.
    - Consequently, the claims we make about populations are always subject to error; hence the term "inferential statistics" and not deductive statistics.
- Inferential statistics encompasses a variety of procedures to ensure that the inferences are sound and rational, even though they may not always be correct.
- Hence in short, inferential statistics enables us to make confident decisions in the face of uncertainty. At best, we can only be **confident** in our statistical assertions, but **never certain of their accuracy**.

Relation between descriptive and inferential statistics

Statistics (="state arithmetic")

Descriptive: describe data

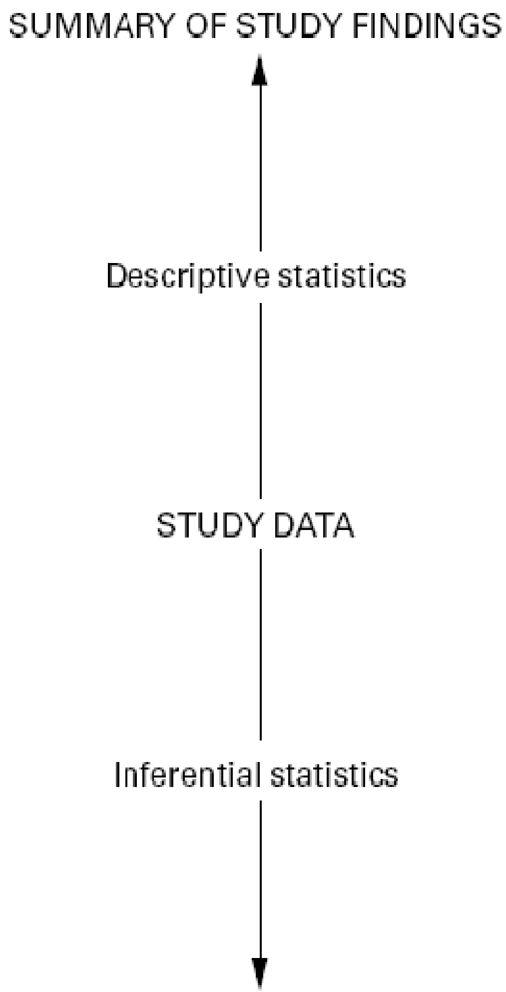
- How rich are our citizens on average? → Central Tendency
- Are there many differences between rich and poor? → Variability
- Are more intelligent people richer? → Association
- How many people earn this money? → Probability distribution
- Tools: tables (all kinds of summaries), graphs (all kind of plots), distributions (joint, conditional, marginal, …), statistics (mean, variance, correlation coefficient, histogram, …)

Inferential: derive conclusions and make predictions

- Is my country so rich as my neighbors? → Inference
- To measure richness, do I have to consider EVERYONE? → Sampling
- If I don't consider everyone, how reliable is my estimate? → Confidence
- Is our economy in recession? → Prediction
- What will be the impact of an expensive oil? → Modelling
- Tools: Hypothesis testing, Confidence intervals, Parameter estimation, Experiment design, Sampling, Time models, Statistical models (ANOVA, Generalized Linear Models, …)

Relation between inductive statistics and deductive statistics

- Whereas **inductive statistics** deal with a limited amount of data, **deductive statistics** involve the logical deduction of the sample properties from knowledge of the population properties (quite similar to interpolation)



GENERALISATION BASED ON STUDY DATA

1.2 Trying to Understand the True State of Affairs

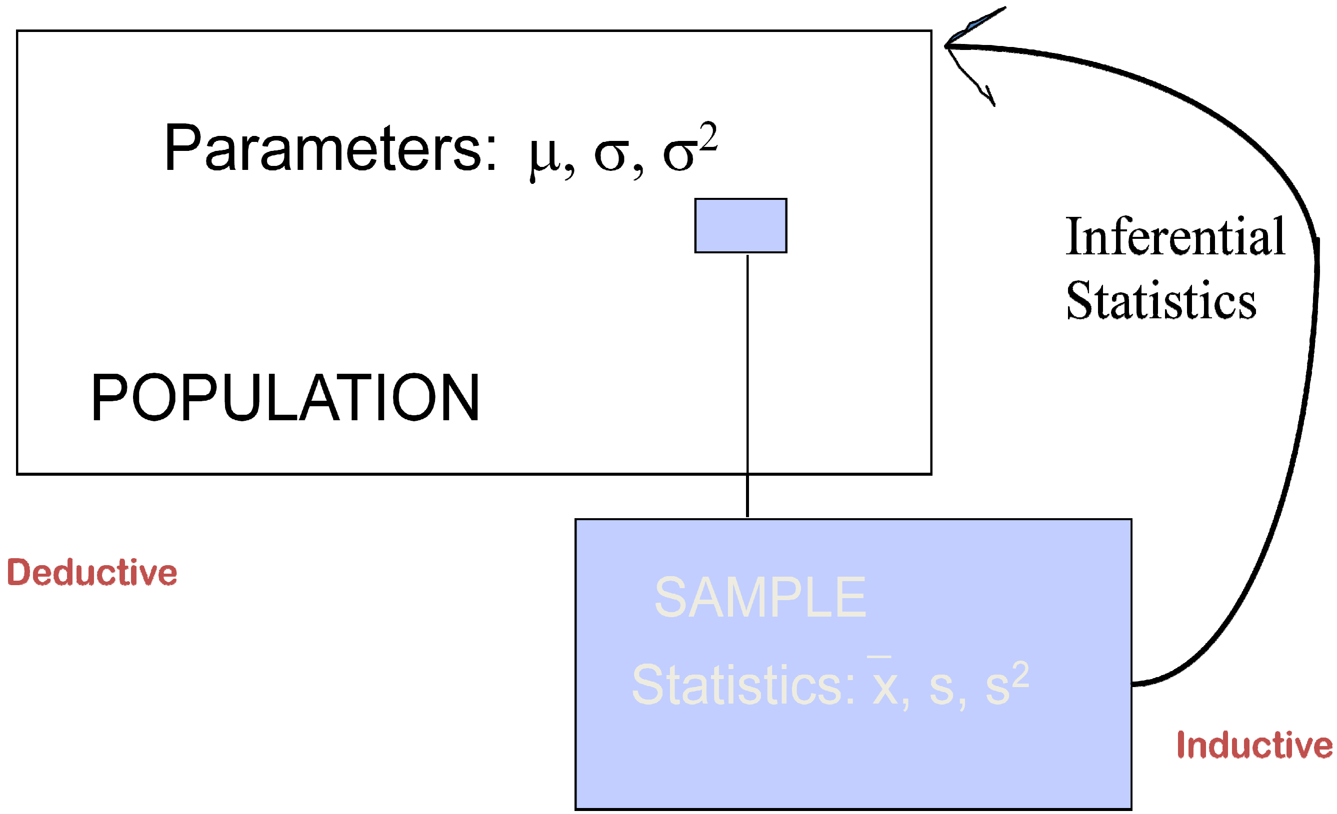
Parameters and statistics

- The world just happens to be a certain way, regardless of how we view it.
- The phrase "**true state of affairs**" refers to the real nature of any phenomenon of interest.
- In **statistics**, the true state of affairs refers to some quantitative property of a population. Numeric properties of populations (such as their means, standard deviations, and sizes) are called **parameters**. Recall from earlier chapters that the parameters of a population (say, its mean and standard deviation) are based on each and every element in that population…
- Samples (or subsets) of populations also have numeric properties, but we call them **statistics**.
- Thus, for the scientist using inferential statistics, population parameters represent the true state of affairs.

- We seldom know the true state of affairs. The process of inferential statistics consists of making use of the data we do have (observed data) to make inferences about population parameters.
- Unfortunately, the true state of affairs is also dependent on all of the data we don't have (unobserved data). Nevertheless, an important aspect of sample data is that they are actual elements from an underlying population. In this way, **sample** data are **'representatives' of the population** that gave rise to them. This implies that sample data can be used to estimate population parameters.
- Therefore, inferential statistics (both estimating and testing components) involve **inductive reasoning**: “from specific towards more general”

Samples and populations

- Since sample data are only representatives, they are not expected to be perfect estimators. Consider that we necessarily lose information about a book when we only read a book review. Similarly, we lack information about a population when we only have access to a subset of that population.
- It would be useful to have some **measure of how reliable** (or representative) our sample data really are. **What is the probability of making an error**?
- Obviously, in order to get a better handle on how representative our data are, we must first consider the sampling process itself: we must first study **how to generate samples from populations**, before we can learn to generalize from samples to populations
- It is in this context that the importance of random and independent sampling begins to emerge.


Parameters: μ, σ, σ²
POPULATION
Inferential
Statistics
Deductive
SAMPLE
Statistics: x̄, s, s²
Inductive

1.3 True state of affairs + Chance = Sample data

- Some elements (say, 'heights') in a population are more frequent than others. These more frequent elements are thus over-represented in the population compared to less common elements (e.g., the heights of very short and very tall individuals).
- The **laws of chance** tell us that it is always possible to randomly select any element in a population, no matter how rare (or under-represented) that element may be in the population**. If the element exists, then it can be sampled, plain and simple.**
- However, the laws of probability tell us that rare elements are not expected to be sampled often, given that there are more numerous elements in that same population. It is the more numerous (or more frequent) elements that tend to be sampled each time a random and independent sample is obtained from the population.

Random and independent samples

- A **sample is random** if all elements in the population are equally eligible to be sampled, meaning that chance, and chance alone, determines which elements are included in the sample.
- A **sample is independent** if the chances of being sampled are not affected by which elements have already been sampled.
- When the sampling process is truly random and independent, samples are expected to reflect the most representative elements of the underlying population.

- Example to illustrate these two ideas
  - Imagine that you are interested in the average age of all university students in the United States.
  - For convenience sake, you decide to randomly select one student from each class offered at your university this term.
  - With respect to the original population of interest (all university students in the U.S.), your sample is not random, because only students at your university are eligible to be sampled.
  - Your sample is also not independent, because once you select a student from a class, no other student in that class has a chance of being sampled.
  - In this case, any claims you make based on your sample cannot be applied to the population you are really interested in. At best, you are only investigating the population of students at one particular university.

Random digits

- Random numbers can be generated in several standard software packages or can be retrieved from already existing tables, such as the one below

Line								
101	19223	95034	05756	28713	96409	12531	42544	82853
102	73676	47150	99400	01927	27754	42648	82425	36290
103	45467	71709	77558	00095	32863	29485	82226	90056
104	52711	38889	93074	60227	40011	85848	48767	52573
105	95592	94007	69971	91481	60779	53791	17297	59335

- How would you use these numbers if I had to select randomly 5 questions from a set of 20?

Important consequence of random and independent sampling

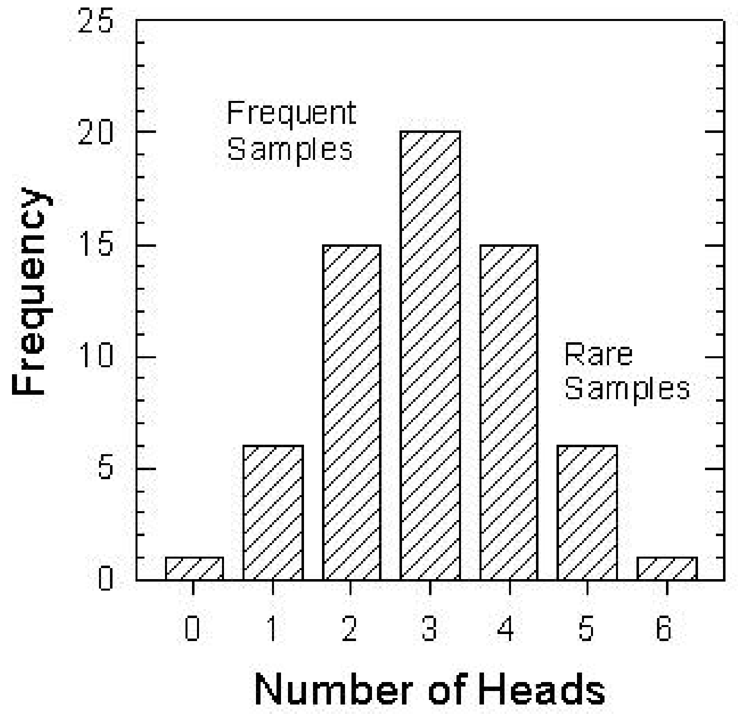
- Chance factors virtually guarantee that sampled data will vary in their degree of representativeness from sample to sample.
- A **rare sample** may occur and occurs when, just by chance, a relatively large number of the extreme (high or low) elements in the population end up in the sample: the percentage of extreme values in the sample is higher than the actual percentage in the population
- The logic is to assume that any particular sample mean is typical of the underlying population. This assumption is reasonable only when the sampling process is random and independent; otherwise, rare samples might artificially occur too often.

Sampling error

- **Sampling error** refers to discrepancies between the statistics of random samples and the true population values; but this "error" is simply due to which elements in the population end up in the sample.
- In other words, sampling error refers to natural chance factors, not to errors of measurement or errors due to poorly designed and poorly executed experiments!!!
- We have control over the execution of an experiment, but nature imparts a certain degree of unavoidable error.

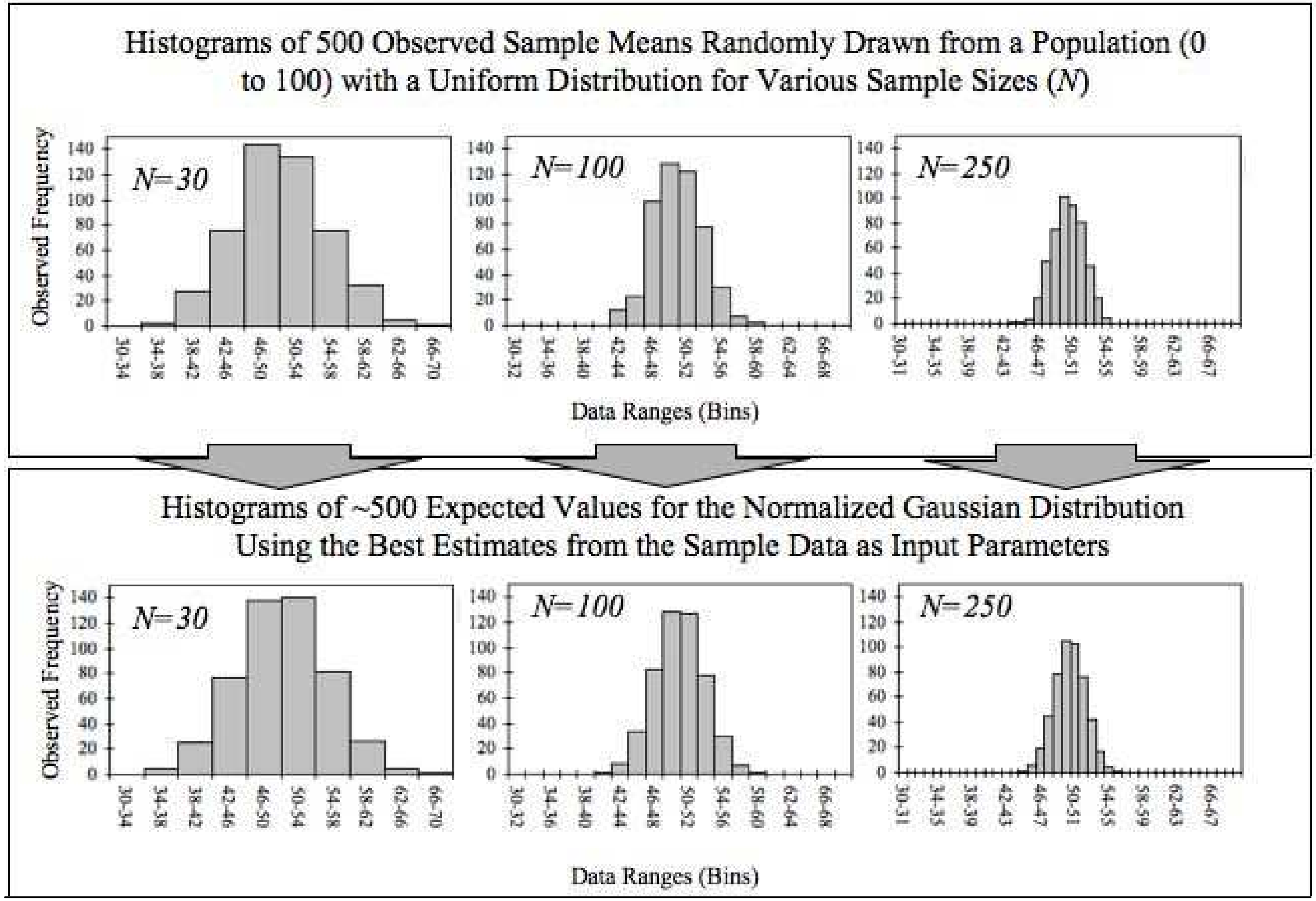
- Illustration of sampling error: tossing a fair coin six times and obtaining {HHHHHH}. We expect a fair coin to land heads 50% of the time, so what went wrong?
  - It turns out there are N = 64 possibilities, but only 20 contain exactly three heads and three tails. In contrast, there is only one outcome containing exactly six heads, which makes it a rare (but not impossible) event.
  - Nonetheless, three heads (in any order) is the most frequent element in this population; it is also the mean.
  - It was because of random sampling that we failed to observe one of these more representative samples, such as {HTHHTT}, not because the mean of the population isn't really 3…

- The laws of chance combined with the true state of affairs create a natural force that is always operating on the sampling process. Consequently, the means of different samples taken from the same population are expected to vary around the 'true' mean just by chance



The Central-Limit Theorem revisited

- A simple example of the central limit theorem is given by the problem of rolling a large number of identical dice, each of which is weighted unfairly in some unknown way. The distribution of the sum (or average) of the rolled numbers will be well approximated by a normal distribution, the parameters of which can be determined empirically.
- In more general probability theory, “a central limit theorem” is any of a set of weak-convergence theories:
  - They all express the fact that a sum of many independent and identically distributed (i.i.d.) random variables, or alternatively, random variables with specific types of dependence, will tend to be distributed according to one of a small set of "attractor" distributions.
  - When the variance of the i.i.d. variables is finite, the "attractor" distribution is the normal distribution.


Histograms of 500 Observed Sample Means Randomly Drawn from a Population (0 to 100) with a Uniform Distribution for Various Sample Sizes (N)
Observed Frequency
N=30
N=100
N=250
Data Ranges (Bins)
Histograms of ~500 Expected Values for the Normalized Gaussian Distribution Using the Best Estimates from the Sample Data as Input Parameters
Observed Frequency
N=30
N=100
N=250
Data Ranges (Bins)

Explanation to the figure

- The sample means are generated using a random number generator, which draws numbers between 1 and 100 from a uniform probability distribution.
- It illustrates that increasing sample sizes result in the 500 measured sample means being more closely distributed about the population mean (50 in this case).
- The input into the normalized Gaussian function is the mean of sample means (~50) and the mean sample standard deviation divided by the square root of the sample size (~28.87/$\sqrt{n}$), i.e. the standard deviation of the mean (since it refers to the spread of sample means).

1.4 Sampling Distributions

- A population is the collection of all possible elements that fit into some category of interest, such as "all adults living in the United States."
- Once we've **defined a population**, we need to specify with respect to what?
  - For instance, all adults living in the United States with respect to their height.
  - Now the population of interest has shifted from a collection of people to a collection of numbers (heights, in this case).
  - When the elements in the population have been measured or scored in some way, it is possible to talk about distributions.
- We can generate a distribution of anything, as long as we have values / scores to work with (cfr tossing a coin and scoring the sample wrt nr of heads).

- When the distribution of interest consists of ***all*** the unique samples of size n that can be drawn from a population, the resulting distribution of sample means is called the **sampling distribution of the mean**.
- There are also sampling distributions of medians, standard deviations, and any other statistic you can think of.
- In other words, populations, which are distributions of individual elements, give rise to sampling distributions, which describe how collections of elements are distributed in the population.
- Note that we have now made a distinction between two distributions:
    - the distribution of individual elements (the population) and
    - the distribution of all unique samples of a particular size from that population (the sampling distribution). [A sample is unique if no other sample in the distribution contains exactly the same elements.]

Level	**Collection**	**Elements**
Population	All individuals (N = size of population)	The scores each individual receives on some attribute.
Sample	Subset of individuals from the population. (n = size of sample)	The scores each individual in the sample receives on some attribute.
Sampling Distribution	All unique samples of size n from the population.	The values of a statistic applied to each sample.

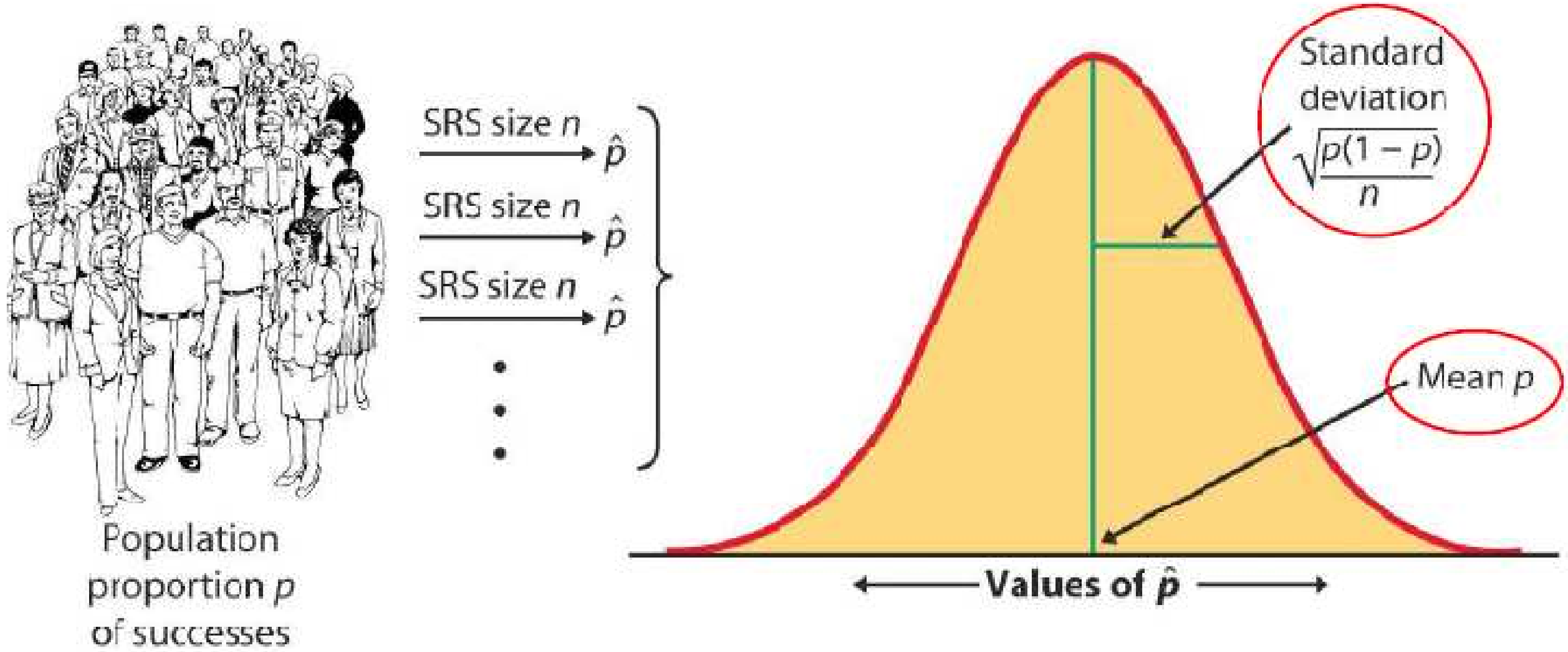
- Sampling distributions are important in inferential statistics, because
  - we obtain samples of data when we conduct studies, and
  - when we are going to make inferences about populations based on sample data, then we need to understand the sampling properties of those samples.

- In inferential statistics we make use of two important properties of sampling distributions (cfr Central Limit Theorem), expressed in lay terms as:
  - The mean of all unique samples of size n (i.e., the average of all the means) is identical to the mean of the population from which those samples are drawn. Thus, any claims about the mean of the sampling distribution apply to the population mean.
  - The shape of the sampling distribution increasingly approximates a normal curve as sample size (n) is increased, even if the original population is not normally distributed.
- If the original population is itself normally distributed, then the sampling distribution will be normally distributed even when the sample size is only one.
  - Can you explain this?

Another example: sampling distribution of the sampling proportion

The sampling distribution of $\hat{p}$ is never exactly normal. But as the sample size increases, the sampling distribution of $\hat{p}$ becomes approximately normal.

The normal approximation is most accurate for any fixed *n* when *p* is close to 0.5, and least accurate when *p* is near 0 or near 1.



Theoretical sampling distributions

- Unless the details of a population are known in advance, it is not possible to perfectly describe any of its sampling distributions.
  - We would have to first measure all the elements in the population, in which case we could simply calculate the desired parameter, and then there would be no point in collecting samples.
- For this reason, a variety of idealized, theoretical sampling distributions have been described mathematically, including the student t distribution or the F distribution (see later, Chapter 5-6), which can be used as statistical models for the real sampling distributions.
- The theoretical sampling distributions can then be used to obtain the likelihood (or probability) of sampling a particular mean if the mean of the sampling distribution (and hence the mean of the original population) is some particular value. The population parameter will first have to be hypothesized, as the true state of affairs is generally unknown. This is called the **null hypothesis** (cfr preview on hypothesis testing + Chapter 6)

Formal definition of a statistic

- Given an independent data set $x_1, x_2, ..., x_n$, let

$$\hat{\theta} = h(x_1, x_2, ..., x_n)$$

  be an estimate of the parameter θ (describing the population from which the sample was drawn).
- Repetitions of the same experiment will give different sets of values for $x_1, x_2, ..., x_n$. Hence, $\hat{\theta}$ itself is a random variable with a certain probability distribution. This distribution will depend on the functional form of h and of the underlying random variable X.
- Therefore, we need to write $\hat{\Theta} = h(X_1, X_2, ..., X_3)$, where $X_1, X_2, ..., X_3$ are random variables, representing a sample from random variable X (and X is in this context referred to as the population).
- Assumptions: All X_i are independent AND $f_{X_i}(x) = f_X(x), \forall x$

Mathematical expression for some popular statistics

- Sample mean: $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
- Sample variance: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n} (X_i - \overline{X})^2$
- k-th sample moment: $M_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

See practicals to derive expectations and variances of these

new random variables ….

Strategies for variance estimation

- Even with a simple random sampling design, the variance estimation of some statistics requires non-standard estimating techniques.
  - For example, the variance of the median is conspicuously absent in the standard texts.
  - The sampling error of a ratio estimator is complicated because both the numerator and denominator are random variables.
- Hence estimating techniques are needed that are sufficiently flexible to accommodate both the complexities of the sampling design and the various forms of statistics.

Method 1: replicated sampling

- The essence of this strategy is to facilitate the variance calculation by selecting a set of replicated subsamples instead of a single sample.
- It requires each subsample to be drawn independently and to use an identical sample selection design.
- Then an estimate is made in each subsample by the identical process,

and the sampling variance of the overall estimate (based on all subsamples) can be estimated from the variability of these independent subsample estimates.

Method 2: Jackknife or bootstrap

- Jackknife and bootstrap approaches are popular when estimating variances and confidence intervals, but is beyond the scope of this introductory course
- The jackknife procedure is to estimate the parameter of interest n times, each time deleting one sample data point. The average of the resulting estimators, called "pseudovalues", is the **jackknife estimate** for the parameter. For large n, the jackknife estimate is approximately normally distributed about the true parameter.
- The bootstrap method involves drawing samples repeatedly from the empirical distribution. So in practice, n samples are drawn with replacement, from the original n data points. Each time, the parameter of interest is estimated from the bootstrap sample, and the average over all bootstrap samples is taken to be the **bootstrap estimate** of the parameter of interest.

Method 3: the Delta method

Taylor Series Expansion: The Taylor series expansion of a function $f(\cdot)$ about a value a is given as:

$$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \cdots,$$

where we can often drop the higher order terms to give the approximation:

$$f(x) \approx f(a) + f'(a)(x-a).$$

Letting $a = \mu_x$, the mean of X, a Taylor series expansion of $y = f(x)$ about μ_x gives the approximation:

$$y = f(x) \approx f(\mu_x) + f'(\mu_x)(x - \mu_x).$$

Taking the variance of both sides yields:

$$\text{Var}(Y) = \text{Var}(f(X)) \approx [f'(\mu_x)]^2 \text{Var}(X).$$

Example: Suppose $Y = X^2$. Then $f(x) = x^2$ and $f'(x) = 2x$, so that:

$$\mathrm{Var}(Y) \approx (2\mu_x)^2 \mathrm{Var}(X) = 4\mu_x^2\sigma_x^2.$$

Example: Suppose $Y = 1/X$. Then $f(x) = 1/x$ and $f'(x) = -1/x^2$, so that:

$$\mathrm{Var}(Y) \approx \left[-\frac{1}{\mu_x^2}\right]^2 \mathrm{Var}(X) = \frac{\sigma_x^2}{\mu_x^4}.$$

Two-Variable Taylor Series Expansion: Suppose now we have random variables X, Y. A Taylor series expansion of $f(x, y)$ about the values (x_0, y_0) is given by:

$$f(x,y) \;=\; f(x_0,y_0) + \left.\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f(x,y)}{\partial y}\right|_{(x_0,y_0)} (y - y_0) + \left(\begin{array}{l}\text{2nd and higher}\\ \text{order terms}\end{array}\right)$$

Example: Suppose $f(x,y) = \frac{y}{x}$. Then: $\frac{\partial f(x,y)}{\partial x} = \frac{-y}{x^2}$, $\frac{\partial f(x,y)}{\partial y} = \frac{1}{x}$

$$\Longrightarrow \quad f(x,y) = \frac{y}{x} \approx \frac{\mu_y}{\mu_x} + \frac{-\mu_y}{\mu_x^2}(x - \mu_x) + \frac{1}{\mu_x}(y - \mu_y)$$

$$\Longrightarrow \quad \mathrm{Var}\left(\frac{Y}{X}\right) \approx \frac{\mu_y^2}{\mu_x^4}\mathrm{Var}(X) + \frac{1}{\mu_x^2}\mathrm{Var}(Y) - \frac{2\mu_y}{\mu_x^3}\mathrm{Cov}(X,Y).$$

(using the fact that the variance of the sum of two random variables is the sum of the variances plus two times the covariance).

Hence the approximate variance of the **ratio estimator** is

$$\mathrm{Var}\left(\frac{\overline{y}}{\overline{x}}\right) \approx \left[\frac{\mu_y^2}{\mu_x^4}\cdot\frac{\sigma_x^2}{n}+\frac{1}{\mu_x^2}\cdot\frac{\sigma_y^2}{n}-\frac{2\mu_y}{\mu_x^3}\cdot\frac{\rho\sigma_x\sigma_y}{n}\right]\cdot \underbrace{\left(\frac{N-n}{N}\right)}_{\substack{\text{if the fpc}\\ \text{is required}}},$$

$$\text{where: } \mathrm{Cov}(\overline{X},\overline{Y}) = \frac{\mathrm{Cov}(X,Y)}{n} = \frac{\rho\sigma_x\sigma_y}{n}.$$

The corresponding **estimated variance** of the ratio estimator is

$$\widehat{\mathrm{Var}}\left(\frac{\overline{y}}{\overline{x}}\right) \approx \frac{1}{n}\left[\frac{\overline{y}^2}{\overline{x}^4}s_x^2+\frac{1}{\overline{x}^2}s_y^2-\frac{2\overline{y}}{\overline{x}^3}\widehat{\rho}s_xs_y\right].$$

Finite population corrections: see section 1.5 of this Chapter

Some Useful Approximations: The linear approximation via a Taylor series expansion gives the approximate variance for the following three useful functions of random variables X and Y where ρ is the correlation between X and Y.

1. $\mathrm{Var}\left(\frac{1}{X}\right) = \left(\frac{1}{\mu_X^4}\right)\sigma_X^2.$
2. $\mathrm{Var}\left(\frac{Y}{X}\right) = \left(\frac{\mu_Y^2}{\mu_X^4}\right)\sigma_X^2 + \left(\frac{1}{\mu_X^2}\right)\sigma_Y^2 - 2\left(\frac{\mu_Y}{\mu_X^3}\right)\rho\sigma_X\sigma_Y.$
3. $\mathrm{Var}(XY) = \mu_Y^2\sigma_X^2 + \mu_X^2\sigma_Y^2 + 2\mu_X\mu_Y\rho\sigma_X\sigma_Y.$
   Note: If X and Y are independent, then an exact expression for $\mathrm{Var}(XY)$ can be derived: $\mathrm{Var}(XY) = \mu_Y^2\sigma_X^2 + \mu_X^2\sigma_Y^2 + \sigma_X^2\sigma_Y^2.$

To obtain estimates of these variances, simply substitute sample values of the means, variances and correlation.

1.5 The Standard Error of the Mean: A Measure of Sampling Error

- Sampling distributions have a standard deviation, which describes the variability of the sample statistics from their mean (which, remember, equals the population parameter).
- Sample size determines both the size and the variability of a sampling distribution
  - The number of unique samples that can be drawn from a population depends on the size of those samples.
  - As sample size increases, the variability among all possible sample means decreases: If all the elements in the original population are sampled (i.e., if $n = N$), then there is only one possible sample that can be obtained (the sample is the population) and the variability of a single number is zero.

- The standard deviation of a sampling distribution of means is given a special name: **standard error of the mean** (often abbreviated as SEM).
- SEM is a measure of sampling error because it describes the variability among all possible means that could be sampled in an experiment.
- If there is a lot of variability in the sampling distribution (as is the case when the distribution consists of small samples), then sample means can vary greatly. On the other hand, if there is little variability in the sampling distribution (as is the case when the distribution consists of large samples), then sample means will tend to be very similar, and very close to the true population mean.
- So the degree of variability in the sampling distribution bears directly on the degree to which observed results (sample means) are expected to vary just by chance.

- **How can we know whether our sample is representative of the underlying population?**
  - Avoid small samples, as there are more extreme (i.e., rare) sample means in the sampling distribution, and we are more likely to get one of them in an experiment.
  - We have control over sampling error because sample size determines the standard error (variability) in a sampling distribution.
  - In Chapter 6, we will see that sample size is closely connected to the concept of power: if a specific power is targeted to identify an effect in a testing strategy, then one can compute the necessary sample size to achieve the pre-specified power of the test.
  - On a practical note: Realize that large samples are not always attainable and that clever more complicated sample strategies than simple random sampling need to be followed.

Correction when sampling from a finite population

- The central limit theorem and the standard errors of the mean and of the proportion are based on the premise that the samples selected are chosen with replacement.
- However, in virtually all survey research, sampling is conducted without replacement from populations that are of a finite size N.
- In these cases, particularly when the sample size n is not small in comparison with the population size N (i.e., more than 5% of the population is sampled) so that $n/N > 0.05$, a **finite population correction factor** (fpc) is used to define both the standard error of the mean and the standard error of the proportion.
- The finite population correction factor is expressed as

$$\sqrt{\frac{N-n}{N-1}},$$

- Therefore, when dealing with means, the standard error of the mean for finite populations is

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

- When referring to populations, the standard error of the proportion for finite populations is

$$\sigma_{p_X} = \sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}$$

- Note that the correction is always smaller than 1, hence reducing the original uncorrected value.
- As a consequence, more precise estimates are obtained after correction …

1.6 Making Formal Inferences about Populations: Preview to Hypothesis Testing

- When there are many elements in the sampling distribution, it is always possible to obtain a rare sample (i.e., one whose mean is very different from the true population mean).
- The probability of such an outcome occurring just by chance is determined by the particular sampling distribution specified in the **null hypothesis**
- When the probability p of the observed sample mean occurring by chance is really low (typically less than one in 20, e.g., $p < .05$), the researcher has an important decision to make regarding the hypothesized true state of affairs.

- One of two inferences can be made:
  1. The hypothesized value of the population mean is correct and a rare outcome has occurred just by chance (as in the coin-tossing example).
  2. The true population mean is probably some other value that is more consistent with the observed data. Reject the null hypothesis in favor of some alternative hypothesis.
- The rational decision is to assume #2, because the observed data (which represent direct, albeit partial, evidence of the true state of affairs), are just too unlikely if the hypothesized population is true.
- Thus, rather than accept the possibility that a rare event has taken place, the statistician chooses the more likely possibility that the hypothesized sampling distribution is wrong.

- However, rare samples do occur, which is why statistical inference is always subject to error.
    - Inferential statistics only helps out to rule out values; it does not tell us what the population parameters really are. We are inferring the values based on what they are likely not to be
    - Only in the natural sciences does evidence contrary to a hypothesis lead to rejection of that hypothesis without error.