## ABEL tutorial

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## Chapter 1

## Overview

GenABEL is an R library developed to facilitate Genome-Wide Association (GWA) analysis of binary and quantitative traits. GenABEL is implemented as an R library. $R$ is a free, open source language and environment for general-purpose statistical analysis (available at http://www.r-project.org/). It implements powerful data management and analysis tools. Though it is not strictly necessary to learn everything about R to run GenABEL, it is highly recommended as this knowledge will improve flexibility and quality of your analysis.

Originally GenABEL was developed to facilitate GWA analysis of quantitative traits using data coming from extended families and/or collected form genetically isolated populations. At the same time GenABEL implements a large number of procedures used in analysis of population-based data; it supports analysis of binary and quantitative tarits, and of survival (time-till-event) data. Most up-to-date information about GenABEL can be found at the web-site http: //mga.bionet.nsc.ru/nlru/GenABEL/.

GenABEL is a part of more extensive ABEL collection (http://mga.bionet. nsc.ru/~yurii/ABEL/) of software supporting different kinds of GWA analyses.

This tutorial was originally written to serve as a set of exercises for the "Advances in population-based studies of complex genetic disorders" (GE03) course of the Netherlands Institute of Health Sciences (Nihes).

If you read this tutorial not as a part of the GE03 course, and you are eager to start with you GWA analysis without reading all the not-so-strictly-necessary staff, start directly from the section 5 ('5').

Otherwise, you can start with R basics and simple association analyises using few SNPs in section $2,,^{2}$ '. In the next section, 4 ( $(\sqrt[4]{4}$ ') you will learn how to work with the gwaa.data-class, which is used to store GWA data in GenABEL and will perform some simple large-scale analyses.

In the next section, $5(\sqrt{5})$, you will do quality control of genetic data and do association analysis under realistic conditions. This section is the core of this tutorial.

The section $\left.7(\sqrt{7}]^{\prime}\right)$ is dedicated to analysis in presence of populational stratification and analysis of family-based data.

Genetic data imputations are covered in section 9 , '9'.
The last section, $\sqrt{12}\left(\cdot \sqrt{12}{ }^{\prime}\right)$, is dedicated to analysis of haplotype association and analysis of SNP interactions.

Information on importing the data from different formats to GenABEL is given in appendix A. Answers to exercises are provided in appendix B

Experienced $R$ users start directly with the section (4, '41').

## Chapter 2

## Introduction to $\mathbf{R}$

In this section we will consider base R data types and operations, and tools for analysis of qualitative and quantitative traits. Only basic R functionality - the things which are crucial to know before we can proceed to genetic association analysis - will be covered within this section. If you want to make most of your data, though, we strongly recommend that you improve your knowledge of $R$ using books other than this. A number of excellent manuals ('An introduction to R', 'Simple R', 'Practical Regression and Anova using R', and others) is available free of charge from the R project web-site (http://www.r-project.org).

In the first part of this chapter you will learn about the most important $R$ data types and will learn how to work with $R$ data. Next, we will cover exploratory data analysis. The chapter will end with introduction to regression analysis.

### 2.1 Basic R data types and operations

On the contrast to many other statistical analysis package, analysis in $R$ is not based on graphic user interface, but is command line-based. When you first start R, a command prompt appears. To get help and overview of R, type help.start() on the command line and press enter. This will start internet browser and open the main page of the R documentation.

Let us first use R as a powerful calculator. You can directly operate with numbers in R. Try multiplying two by three:

```
> 2 * 3
```

[1] 6
Other standard arithmetic operations can be performed in similar manner:
$>2 / 3$
[1] 0.6666667
(division)

## [1] 8

(power)
> 2 - 3
[1] -1
(subtraction)
$>2+3$
[1] 5
(summation) ${ }^{1}$.
Mathematical functions, such as square roots, base-10 logarithm, and exponentiation, are available in R as well:
$>\operatorname{sqrt}(5)$
[1] 2.236068
> log10(2.24)
[1] 0.350248
$>\exp (0.35)$
[1] 1.419068
Here, we have computed $e$ to the power of base-10 logarithm of the square root of the sum of two and three. After each operation performed, we have rounded the result to the two digits after the floating point - just in order to do less typing.

The arithmetic operations and functions can be nested and therefore we can obtain the above result in one line, and without the 2nd-digit approximation:

```
> exp(log10(sqrt(2 + 3)))
```


## [1] 1.418337

R functions include not only the standard mathematical ones, but also a wide range of statistical function, for example, probability density functions of many probability distributions. We will make extensive use of these at a later stage, when computing significance and estimating statistical power.

For any function with name say 'fun', help may be obtained by typing 'help(fun)' on the command line.
$R$ help pages have standard layout, documenting usage of the function, explaining function arguments, providing details of implementation and/or usage, explaining the value returned by the function, and giving references and examples of the function use.

Exercise 1. Explore the help page for the Wilcoxon test and answer the questions:

[^0]1. When exact Wilcoxon test is computed by default?
2. If the default conditions for the exact test are not satisfied, what approximation is used?

Most of the documented functions have examples of their usage at the end of the 'help' page, and these examples can be evaluated in R. Try 'example (wilcox.test)'.

If you do not know the exact name for the function you look for, try 'help.search("query")', where query is the keyword.

Exercise 2.Try to find out what are the functions to do

1. Fisher exact test
2. T-test

One of important R operations is assignment, which is done with '<-' operator. A (new) variable name should be provided on the left-hand side of this operator and on the right-hand side, there must be either name of already existing variable or an expression. For example, we if want to assign value ' 2 ' to variable ' $a$ ', and value ' 3 ' to the variable ' $b$ ' we would use the assignment operator:

```
> a <- 2
> b <- 3
```

Typing the variable name in R command line will return its' value, e.g.

```
> b
```

[1] 3
Evaluation of the expression
> $\exp (\log 10(\operatorname{sqrt}(a+b)))$
[1] 1.418337
gives the expected result we have obtained above using numerical arguments.
While the variables 'a' and 'b' contain single numeric values, variables in general can be multi-dimensional; an one-dimensional example of such is a vector (array). Let us create an example vector and experiment with it:

```
>v <- c(1, 3, 5, 7, 11)
```

Here, 'c ()' is a function, which combines its arguments to make a vector. This vector is then assigned to a variable named 'v'.

Now, let us try different operations with this vector:
$>v+1$
[1] $2 \begin{array}{lllll}2 & 4 & 6 & 8 & 12\end{array}$
It is easy to see that the result is a vector, which is obtained by adding one to each element of the original vector v . Other arithmetic operations and mathematical functions behave in the same way, e.g. the operation is performed for each element of the vector, and the results are returned:
> $1 / \mathrm{v}$
[1] 1.00000000 .33333330 .20000000 .14285710 .0909091
$>\log (v)$
[1] 0.0000001 .0986121 .6094381 .9459102 .397895
What happens if two vectors are supplied as function arguments? Let us define a new vector

```
> ov <- c(1, 2, 3, 4, 5)
```

and add it to the vector v :

```
> v + ov
```

[1] $2 \begin{array}{lllll}5 & 8 & 11 & 16\end{array}$
You can see that the summation was done element-wise, i.e. the first element of the result vector is obtained as the sum of the first elements of $v$ and ov, the second is the sum of the second elements, and so forth.

Other arithmetic operations with two vectors are performed in the same element-wise manner:

```
> v * ov
```

[1] $1 \begin{array}{lllll}1 & 6 & 15 & 28 & 55\end{array}$
(multiplication)
> v^ov

| [1] | 1 | 9 | 125 | 2401 | 161051 |
| :--- | :--- | :--- | :--- | :--- | :--- |

(power).
The vector operations considered above returned a same-length vector as output. There are others - statistical and summary - functions which evaluate a vector as a whole and return a single value as output. For example, to obtain a sum of vector's elements, use

```
> sum(v)
```

[1] 27
Other examples of such functions involve length, returning number of elements of a vector, mean, returning the mean, var, returning the variance, etc.:

```
> length(v)
```

[1] 5
> mean(v)
[1] 5.4
> $\operatorname{var}(\mathrm{v})$
[1] 14.8
One of the basic, and probably most used, data operations in R is sub-setting. This refers to an operations which help you deriving a subset of the data. Let us create a short vector and play a bit with sub-setting. This vector will contain 5 simple character strings:

```
> a <- c("I am element 1", "I am element 2", "I am element 3",
+ "I am element 4", "I am element 5")
>a
[1] "I am element 1" "I am element 2" "I am element 3" "I am element 4"
[5] "I am element 5"
```

To find out what is the value of the $i$-th element of this vector, you can sub-set it by a[i]. For example the 3rd elements is:
> a[3]
[1] "I am element 3"
You can also select a bigger sub-set, e.g. all elements from 2 to 4:
$>a[c(2: 4)]$
[1] "I am element 2" "I am element 3" "I am element 4"
Here, operation $c(2: 4)$ stays for 'combine numbers from 2 to 4 into a vector'. An equivalent result is obtained by

```
> a[c(2, 3, 4)]
```

[1] "I am element 2" "I am element 3" "I am element 4"
We can also easily get disjoint elements; e.g. if you want to retrieve elements 1,3 , and 5 , you can do that with
> dje <- c(1, 3, 5)
> dje
[1] 135
> a[dje]
[1] "I am element 1" "I am element 3" "I am element 5"
One of very attractive features of R data objects is possibility to derive a subset based on some condition. Let us consider two vectors, tmphgt, containing the height of some subjects, and tmpids, containing their identification codes (IDs):

```
> tmphgt <- c(150, 175, 182, 173, 192, 168)
> tmphgt
```

```
[1] 150 175 182 173 192 168
> tmpids <- c("fem1", "fem2", "man1", "fem3", "man2", "man3")
> tmpids
[1] "fem1" "fem2" "man1" "fem3" "man2" "man3"
```

Imagine you need to derive the IDs of the people with height over 170 cm . To do that, we need to combine several operations. First, we shoudl run the logical function >170 on the height data:

```
> vec <- (tmphgt > 170)
> vec
```


## [1] FALSE TRUE TRUE TRUE TRUE FALSE

This returns a logical vector whose elements are 'TRUE', when a particular element of the tmphgt satisfies the condition $>170$. The returned logical vector, in turn, can be applied to sub-set any other vector of the same length ${ }^{2}$ including itself. Thus if you need to see what are the heights in people, which are taller than 170 cm , you can use

```
> tmphgt[vec]
```


## [1] 175182173192

As you can see, only the elements of tmphgt, for which the corresponding value of vec was 'TRUE', are returned. In the same manner, the logical vector vec can be applied to select elements of the vector of IDs:

```
tmpids[vec]
```

```
[1] "fem2" "man1" "fem3" "man2"
```

You can combine more than one logical condition to derive sub-sets. For example, to see what are the IDs of people taller than 170 but shorter than 190 cm , you can use

```
> vec <- (tmphgt > 170 & tmphgt < 190)
> vec
[1] FALSE TRUE TRUE TRUE FALSE FALSE
> tmpids[vec]
[1] "fem2" "man1" "fem3"
```

A better ${ }^{3}$ way to do logical sub-setting assumes use of the which() function on the top of the logical vector. This function reports which elements are TRUE. To obtain above results you can run:

[^1]```
> vec <- which(tmphgt > 170 & tmphgt < 190)
>vec
```

[1] 234
> tmpids[vec]

```
[1] "fem2" "man1" "fem3"
```

You can see that no vec contains a vector, whose elements are the indexes of the elements of tmphgt for which the logical condition satisfies.

Sub-setting for 2D objects (matrices) is done in similar manner. Let us construct a simple matrix and do several sub-setting operations on it:

```
> a <- matrix(c(11, 12, 13, 21, 22, 23, 31, 32, 33), nrow = 3,
+ ncol = 3)
> a
```

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ |
| :--- | ---: | ---: | ---: |
| $[1]$, | 11 | 21 | 31 |
| $[2]$, | 12 | 22 | 32 |
| $[3]$, | 13 | 23 | 33 |

To obtain the element in the 2nd row and 2nd column, you can use
$>a[2,2]$
[1] 22
To access the elemnt from the second row and third column, use

```
> a[2, 3]
```

[1] 32
Note that here, the row index (2) comes first, and the column index (3) comes second.

To obtain the 2 x 2 set of elements contained in upper left corner, you can do $>a[1: 2,1: 2]$

|  | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| $[1]$, | 11 | 21 |
| $[2]$, | 12 | 22 |

Or you can even get the variables, which reside in corners:
$>a[c(1,3), c(1,3)]$

|  | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| $[1]$, | 11 | 31 |
| $[2]$, | 13 | 33 |

If one of the dimensions is not specified, complete vector is returned for this dimension. For example, here we retrieve the first row

|  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| 1 | 1 | 4 | 7 |
| 2 | 2 | 5 | 8 |
| 3 | 3 | 6 | 9 |

Table 2.1: Vector representation of a matrix. Elements in the table are the vector indices of the matrix elements.
> $a[1$,
[1] 112131
...and the third column
> $a[, 3]$
[1] 313233
...or columns 1 and 3:
$>a[, c(1,3)]$

|  | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| $[1]$, | 11 | 31 |
| $[2]$, | 12 | 32 |
| $[3]$, | 13 | 33 |

Other way to address elements of a matrix is to use one-dimensional index. For example, if you want to access element in the 2nd row and 2 nd column, instead of
$>a[2,2]$
[1] 22
you can use
> a[5]
[1] 22
This way of accessing the elements of a matrix is based on the fact, that each matrix can be preseted as a vector, whose elements are numbered consequtively: the element in the upper-left corner has index 1 , the element in the second row of the first column has index 2, and the last elemnt in the borrom-right corner has the maximal value, as shown in Table 2.1.

As well as with vectors, you can sub-set matrices using logical conditions or indexes. For example, if we want to see what elements of a are greater than 21, we can run

```
    [,1] [,2] [,3]
[1,] FALSE FALSE TRUE
[2,] FALSE TRUE TRUE
[3,] FALSE TRUE TRUE
or, better
> which(a > 21)
[1] 56789
```

Note that in the latter case, a vector whose elements give the 1-D indexes of the matrix, is returned. This vector indicates the elemnts of matrix a, for which the condition ( $a>21$ ) is satisfied.

You can obtain the values of the matrix's elements, for which the condition isfulfilled, either by
> a[a > 21]
[1] $22 \quad 23 \quad 31 \quad 3233$
or
> a[which(a > 21)]
[1] $22 \quad 23313233$
Once again, the latter method should be prefered. Consider an example, where some elements of the matrix are missing (NA) - a situation which is common in real data analysis. Let us replace the elemnt number 5 with NA and perform sub-setting operations on the resulting matrix:
> a

```
    [,1] [,2] [,3]
[1,] 11 21 31
[2,] }12 22 3
[3,] 13 23 33
>a[5] <- NA
> a
\begin{tabular}{lrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} \\
{\([1]\),} & 11 & 21 & 31 \\
{\([2]\),} & 12 & NA & 32 \\
{\([3]\),} & 13 & 23 & 33
\end{tabular}
> a[a > 21]
[1] NA 23 31 32 33
> a[which(a > 21)]
[1] 23 31 32 33
```

You can see that when $a[a>21]$ was used, not only the elements which are greater than 21 were returned, but also NA was. As a rule, this is not what you want, and which should be used unless you do want to make some use of the NA elements.

In this section, we have generated a number of R data objects. Some of these were numeric (e.g. vector of heights, tmphgt) and some were character, or string (e.g. vector of study IDs, tmpids). Some times you need to figure out what is the class of a certain object. This can be done using the class() function. For example,

```
> tmphgt
[1] }15
> class(tmphgt)
[1] "numeric"
> tmpids
[1] "fem1" "fem2" "man1" "fem3" "man2" "man3"
> class(tmpids)
[1] "character"
```

What happens if we try to find out the class of
$>\mathrm{a}$

|  | $[, 1]$ | $[, 2]$ | $[, 3]$ |
| :--- | ---: | ---: | ---: |
| $[1]$, | 11 | 21 | 31 |
| $[2]$, | 12 | NA | 32 |
| $[3]$, | 13 | 23 | 33 |

- an object, which contains a matrix?
> class(a)
[1] "matrix"
Results are expected - we find out that a is a matrix, which is correct. At the same time, a matrix is an upper-level class, which contains a number of elemnts, belonging to some lower-level (e.g. character/numeric) class. To see what is the class of the matrix's elements, try
$>a[1$,
[1] 112131
> class (a[1, ])
[1] "numeric"
which says that elemnts (at least of the first row) are numeric. Because all elements of a matrix should have the same class, we can conclude that a is a matrix containing numeric values.

At this point, it is worthwile inspecting what data objects were created during our work. This can be done with the ls() command:
> ls()
[1] "a" "b" "dje" "ov" "tmphgt" "tmpids" "v" "vec"

Obviously, this "list" command is very useful - you will soon find that it is just too easy to forget the name of a variable which it took long time to create. Some times you may wish to remove some of the data objects because you do not need then anymore. You can remove an object using the rm() command, where the names of objects to be deleted are listed as arguments. For example, to remove tmphgt and tmpids variable you can use

```
> rm(tmphgt, tmpids)
```

If you now look up what data obejcts are still left in you workspace with the ls() command
$>\operatorname{ls}()$
[1] "a" "b" "dje" "ov" "v" "vec"
you find that you have successfully deleted tmphgt and tmpids.
At this point, you can exit $R$ by typing $q()$ on the command line and pressing Enter.

## Summary:

- You can get access to the top-level R documentation by help.start() command. To search help for some keyword keywrd, you can use help.search (keywrd) command. To get description of some function fun, use help (fun).
- You can use R as a powerful calculator
- It is possible to get sub-sets of vectors and matrices by specifying index value or a logical condition (of the same length as the vector / matrix) between square brackets ([, ])
- When you obtain an element of a matrix with $[i, j]$, $i$ is the row and $j$ is the column of the matrix.
- Function which(A) returns index of the elements of A which are TRUE
- You can see objects available in your workspace by using the ls() command
- Unnecessary object (say, tmphgt) can be deleted from the workspace using rm command, e.g. rm (tmphgt)
- You can leave R using the q() command


## Exercise 1.

In this exercise, you will explore few vectors representing different data on study subjects described in srdta example data set supplied together with GenABEL . First, you need to load GenABEL by typing
> library (GenABEL)
and load the data by
>data(srdta)
The vector containing study subjects sex can be accessed through srdta@gtdata@male; this vector's value is one when the corresponding person is male and zero otherwise. The vector containing SNP names can be accessed via srdta@gtdata@snpnames, chromosome ID - through srdta@gtdata@chromosome and map - through srdta@gtdata@map. Explore these vectors and answer the questions.

1. What is the ID and sex of the first person in the data set?
2. Of the 22 nd person?
3. How many males are observed among first hundred subjects?
4. How many FEMALES are among 4th hundred?
5. What is the male proportion in first 1000 people?
6. What is the FEMALE proportion in second 1000 (1001:2000) people?
7. What is name, chromosome and map position of 33 rd maker?
8. What is distance between markers 25 and 26 ?

### 2.2 Data frames

A data frame is a class of R data, which, basically, is a data table. In such tables, it is usually assumed that rows correspond to subjects (observations) and columns correspond to variables (characteristics) measured on these subjects. A nice feature of data frames is that columns (variables) have names, and the data can be addressed by referencing to these names ${ }^{4}$

We will explore R data frames using example data set assoc. Start R with double-click on the file named assocbase. RData. You can see the names of the loaded objects by using the "list" command:
> ls()
[1] "assoc"
Thus, only one object is loaded. The class of this object is:

```
> class(assoc)
```

[1] "data.frame"

- a data frame.

The dimensionality of a data frame (or a matrix) can be determined by using the $\operatorname{dim}()$ command:
$>\operatorname{dim}(\mathrm{assoc})$

[^2][1] 2507
Here, the first number corresponds to the number of rows (subjects) and the second to the number of columns (variables). Thus, the data frame assoc contains the data on 250 subjects, who are characterised by 7 variables each.

Let us now figure out what are the names of the 7 variables present in the data frame. To see what are the variable names, use the command names():

```
> names(assoc)
```

[1] "subj" "sex" "aff" "qt" "snp4" "snp5" "snp6"
These variables correspond to the personal identifier (ID, variable subj), sex, affection status, quantitative trait qt and several SNPs. Each variable can have its own type (numeric, character, logic), but all variables must have the same length - thus forming a matrix-like data structure.

A variable from a data frame (say, fram), which has some name (say, nam) can be accessed through fram\$nam. This will return a conventional vector, containing the values of the variable. For example to see the affection status (aff) in the data frame assoc, use

```
> assoc$aff
```

| $[1]$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $[26]$ | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | $N A$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $[51]$ | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $[76]$ | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | $N A$ | 0 | 0 | 0 | 0 |
| $[101]$ | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| $[126]$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | $N A$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $[151]$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $[176]$ | 0 | 0 | 1 | 1 | $N A$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | NA | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $[201]$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $[226]$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

The aff (affected) variable here codes for a case/control status, conventinally, the cases are coded as " 1 " and controls as " 0 ". You can also see several "NA"s, which stays for missing observation.

## Exercise 2.

Investigate types of the variables presented in data frame assoc. For each variable, write down the class.

Data frame may be thought of as a matrix which is a collection of (potentily different-type) vectors. All sub-setting operations discussed before for matrices are applicable to a data frame, while all operations dicussed for vectors are applicable to data frame's variables.

Thus, as any particular variable present in a data frame is a conventional vector, its elements can be accessed using the vector's indices. For example, if you would like to know what are the ID, sex and affection status for the person with index 75 , you can request

```
> assoc$subj[75]
```

[1] 1409

```
> assoc$sex[75]
```

[1] 1
> assoc\$aff[75]
[1] 0
Alternatively, using the matrix-style of sub-setting, you can see all the data for person 75 :

```
> assoc[75, ]
    subj sex aff qt snp4 snp5 snp6
75 1409 1 0 1.014664 A/B B/A B/B
```

In the same manner as with matrices, you can get data for e.g. subjects 5 to 15 by

| > assoc[5:15, ] |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | subj | sex | aff | qt | snp4 | snp5 | snp6 |
| 5 | 1533 | 0 | 0 | 0.1009220 | A/B | B/A | B/A |
| 6 | 2466 | 1 | 0 | -0.1724321 | A/B | A/A | A/A |
| 7 | 2425 | 0 | 0 | -0.3378473 | B/B | A/A | A/A |
| 8 | 1068 | 0 | 0 | -1.7112925 | A/A | B/B | <NA |
| 9 | 198 | 1 | 0 | -0.4815822 | A/B | B/A | B/A |
| 10 | 1496 | 1 | 0 | 1.2281232 | A/A | B/B | B/B |
| 11 | 909 | 0 | 0 | 0.5993945 | A/B | B/A | B/A |
| 12 | 1213 | 0 | 0 | 1.9792190 | A/A | B/B | B/B |
| 13 | 181 | 1 | 0 | 1.5435921 | A/A | B/B | B/B |
| 14 | 1783 | 0 | 0 | -1.6242738 | A/B | B/A | B/A |
| 15 | 1914 | 0 | 0 | -0.5160331 | A/A | B/B | B/B |

The result is actually a new data frame containing data only on people with index from 5 to 15 :

```
> x <- assoc[5:15, ]
class(x)
[1] "data.frame"
> dim(x)
[1] 11 7
```

As well as with matrices and vectors, it is possible to sub-set elements of a data frame based on (a combination of) logical conditions. For example, if you are interested in people who have the qt values over 1.4, you can find out what are the indices of these people

```
> vec <- which(assoc$qt > 1.4)
> vec
```

```
[1] 12 13 13 33 41 54 68 72 76 709 106 118 142 156 161 175 181 193 219 241
```

and then show the compelte data with

```
> assoc$subj[vec]
```


[16] $1628 \quad 562 \quad 858 \quad 698$

At the same time, if you only want to check what are the IDs of these people, try

```
> assoc$subj[vec]
```

[1] $1213 \quad 181 \quad 17371319 \quad 516 \quad 1355 \quad 186 \quad 1426 \quad 1284$
[16] $1628 \quad 562 \quad 858 \quad 698$
Or, if we are interested to find what are the IDs and what are the SNP genotypes of these people, we can try

| > assoc[vec, c(1, | 5, 6,7 |  |  |  |
| :--- | ---: | :--- | :--- | :--- |
| subj |  |  |  |  |
| snp4 | snp5 | snp6 |  |  |
| 12 | 1213 | A/A | B/B | B/B |
| 13 | 181 | A/A | B/B | B/B |
| 33 | 1737 | A/A | B/B | B/B |
| 41 | 1319 | A/A | B/A | B/A |
| 54 | 516 | A/B | B/A | B/A |
| 68 | 1355 | A/A | B/B | B/B |
| 72 | 186 | A/A | B/A | B/A |
| 76 | 1426 | A/B | B/A | B/A |
| 89 | 1284 | A/A | B/B | B/B |
| 106 | 822 | A/B | B/A | B/A |
| 118 | 2129 | A/B | B/A | B/A |
| 142 | 212 | A/B | B/A | B/A |
| 156 | 1443 | A/A | B/B | B/B |
| 161 | 704 | A/B | B/A | B/A |
| 175 | 1648 | A/B | B/A | B/A |
| 181 | 1628 | A/B | B/A | B/A |
| 193 | 562 | A/A | B/B | B/B |
| 219 | 858 | A/B | B/A | B/A |
| 241 | 698 | B/B | A/A | A/A |

here, we select people identified by vec in the first dimension (subjects), and by $c(1,5,6,7)$ we select first, fifth, sixth and seventh column (variable).

The same result can be obtained using variables' names insted of the variables' indices. To remind you the variables' names:

```
> names(assoc)
```

[1] "subj" "sex" "aff" "qt" "snp4" "snp5" "snp6"
And now make a vector of the variables' names of interest and filter the data based on it:

```
> namstoshow <- c("subj", "snp4", "snp5", "snp6")
> assoc[vec, namstoshow]
    subj snp4 snp5 snp6
    1213 A/A B/B B/B
    181 A/A B/B B/B
    1737 A/A B/B B/B
    1319 A/A B/A B/A
    516 A/B B/A B/A
    1355 A/A B/B B/B
    186 A/A B/A B/A
76 1426 A/B B/A B/A
89 1284 A/A B/B B/B
106 822 A/B B/A B/A
118 2129 A/B B/A B/A
142 212 A/B B/A B/A
156 1443 A/A B/B B/B
161 704 A/B B/A B/A
175 1648 A/B B/A B/A
181 1628 A/B B/A B/A
193 562 A/A B/B B/B
219 858 A/B B/A B/A
241 698 B/B A/A A/A
```

A more convenient way to access data presented in a data frame is through "attaching" it to the R search path by

```
> attach(assoc)
```

After that, the variables can be accessed directly, e.g.
> subj[75]
[1] 1409
instead of assoc\$subj[75].
While it is possible to explore the data presented in a data frame using the sub-setting operations and screen output, and modify certain data elements using the assignment ("<-") operation, you can also explore and modify the data contained in a data framf ${ }^{5}$ by using fix() command (e.g. try fix(assoc)). However, normally this is not necessary.

With attached data frames, a possible complication is that later on you may have several data frames which contain the variables with the same names. The variable which will be used when you directly use the name would be the one from the data frame attached last. You can use detach() function to remove a certain data frame from the search path, e.g. after

```
> detach(assoc)
```

we can not use direct reference to the name (try subj [75]) anymore, but have to use the full path instead:

[^3]```
> assoc$subj[75]
```

[1] 1409

## Summary:

- The list of available objects can be viewed with 1s(); a class of some object obj can be interrogated with class (obj).
- Simple summary statistics for numeric variables can be generated by using summary function
- Histogram for some variable var can be generated by hist(var)
- A variable with name name from a data frame frame, can be accessed through frame\$name.
- You can attach the data frame to the search path by attach(frame). Then the variables contained in this data frame may be accessed directly. To detach the data frame (because, e.g., you are now interested in other data frame), use detach (frame).


## Exercise 3.

Load the srdta data object supplied with GenABEL by loading the package with library (GenABEL) and then loading the data with data(srdta). The srdta object contains a data frame with phenotypes. This data frame may be accessed through srdta@phdata. Explore this data frame and answer the questions

1. What is the value of the 4 th variable for the subject number 75 ?
2. What is the value of variable 1 for person 75 ? Check what is the value of this variable for the first ten people. Can you guess what first variable is?
3. What is the sum of variable 2? Can you guess what data variable 2 contains?

### 2.3 Exploratory analysis of qualitative and quantitative traits

Let us now attach the data frame asscoc
> attach (assoc)
and explore it.
Let us first check how many of the subjects are males. In the sex variable, males are coded with " 1 " and females with "0". Therefore to see the numer of males, you can use
$>\operatorname{sum}(\operatorname{sex}==1)$
[1] 129
and to determine what is male sex proportion you can use

```
sum(sex == 1)/length(sex)
```

[1] 0.516
This way to compute the proportion would only work correctly if there are no missing observations (lenght () returns the total length of a variable, including NAs).

Because of the way the males are coded, the same answer is reached by

```
> mean(sex)
```

[1] 0.516
However, that would not have worked if the sex was coded differently, e.g. with " 1 " for males and " 2 " for females.

Let us now try to find out the mean of the quantitative trait qt. By definition, the mean of a variable, say $x$ (with i-th element denoted as $x_{i}$ ) is

$$
\bar{x}=\frac{\sum_{i=1}^{N} x_{i}}{N}
$$

where $N$ is the number of measurements.
If we try to find out the mean of qt by direct use of this formula, we first need to find out the sum of the qt's elements. The sum() function of R precisely does the operation we need. However, if we try it
$>\operatorname{sum}(q t)$

## [1] NA

this returns "NA". The problem is that the qt variable contains "NA"s (try qt to see these) and by default the "NA" is returned. We can, however, instruct the sum() function to remove "NA"s from consideration:

```
sum(qt, na.rm = T)
```


## [1] -26.4733

where na.rm=T tells $R$ that missing variables should be be removed (NonAvailable.ReMove $=$ True) ${ }^{6}$

We can now try to compute the mean with

```
> sum(qt, na.rm = T)/length(qt)
```


## [1] -0.1058932

This result, however, is not correct. The length() function returns the total length of a vector, which includes "NA"s as well. Thus we need to compute the number of the qt's elements, which are not missing.

For this, we can use $R$ function is.na(). This function returns TRUE if supplied argument is missing (NA) and FALSE otherwise. Let us apply this function to the vector assoc\$qt:

[^4]
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```
> is.na(qt)
```

[1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [13] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE [25] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [37] TRUE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE [49] FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [61] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [73] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [85] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [97] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [109] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [121] FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE [133] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [145] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [157] FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE FALSE FALSE [169] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [181] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [193] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE [205] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [217] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [229] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE [241] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE

Indeed, the 7 missing elements are correctly identified. However, we are interested in elements which are not missing. To get these, we can use the logical function NOT (!), which changes all FALSE to TRUE and visa versa:

```
> !is.na(qt)
```

|  | TRUE | TRUE | TRUE | TRUE | JE | TRUE | TRUE | TRUE | TR | TRUE | TRUE | TRUE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [13] | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | FA | TRUE | TRUE | TRUE |
| [25] | TR | TRUE | TRUE | TRUE | TRUE | TRUE | TR | TRUE | TRUE | TRUE | TRUE | TRUE |
| [37] | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | FAL | TRUE | TRUE | TRUE | TRUE | TRUE |
| [49] | TRUE | FALSE | TRUE | UE | TRUE | TRUE | RU | RUE | TRU | RUE | RUE |  |
|  | TRUE | RUE | RU | RUE | TRUE | RU | TRU | TRU | TRU | RU | RUE |  |
|  | TR |  |  |  |  |  |  |  | TRU | TRU |  |  |
|  | TRUE | TRUE |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| 109] | TR | TRUE | TRU | TRUE | TRUE | TRUE | TRUE | TRU | TRUE | TRUE | TRUE |  |
| 121] | TR | TRUE |  | TRU | TRUE | TRU | RU | FALS | TRUE | TRU | TRUE |  |
| 13 | TR | TRUE |  | TRUE | TRUE | TRU | TRU | TRUE | TRUE | TRUE | TRUE |  |
|  | TR | TRU |  | TRUE | TRUE | TR | TRUE | TRUE |  | TRUE | TRUE |  |
|  | TRUE | TRUE |  |  | TRUE | FALS |  | TRU | TRUE | RU | RUE |  |
| [169] | TRU | RU | TRUE | TRUE | TRU | RU | RU | TRU | TRU | RU | TRUE |  |
| 181] | TR | TRUE | TRUE | TRUE | TRUE | TR | TR | TR | TRU | TRU | TRUE |  |
| [193] | TR | TR | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | FAL | TRUE |  |
| [205] | TR | TR | TR | TRUE | TRUE | TRUE | TRUE | TR | TR | TRUE | TRUE |  |
| [ | TR | TR | TR | TRUE | TR | TR | TR | TR | TRUE | TRUE | TRUE |  |
| [29] | TRUE | TRUE | TRUE | TRUE | TRUE | TR | TRU | TRU | TRUE | TRUE | TRUE |  |
| $1]$ | TRU | TR | TRU | TRUE | TRUE | TR | R | R |  |  |  |  |

Thus the number of elements which are not missing ${ }^{7}$ is
> sum(!is.na(qt))
[1] 243
Finally, we can compute the mean of the qt with
$>\operatorname{sum}(q t, n a \cdot r m=T) / \operatorname{sum}(!i s . n a(q t))$
[1] -0.1089436
While this way of computing the mean is enlightening in the sense of how to treat the missing values, the same correct result should be normally achieved by supplying the na.rm=T argument to the mean() function:

```
> mean(qt, na.rm = T)
```

[1] -0.1089436
The function table(x) produces a frequency table for the variable x . Thus, we can use

```
> table(sex)
sex
    0}
121129
```

which, again, tells us that there are 129 males and 121 females in this data set. This function excludes missing observations form consideration.

Tables of other qualitative variables, such as affection and SNPs, can be generated in the same manner.

As with arithmetic operations and mathematical functions, most of the $R$ operations can be combined within a single line. Let us try to combine logical conditions and the table() command to check the distribution of number of affected in men and women separately:

```
> table(aff[which(sex == 1)])
    0}
9531
> table(aff[which(sex == 0)])
    0}
95 24
```

On R command line pressing the "up-arrow" button makes the last typed command re-appear (pressing it one more time will bring you to the one before the last, so on). This is very handy when you have to repeat the same analysis of different variables

[^5]
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## Exercise 4.

Explore qualitative variables presented in assoc

1. How many affected and unaffected are present in the data set?
2. What is the proportion of affected?
3. What is the distribution of snp4 (how many different genotype classes are present and what are the counts)?
Contingency tables for pairs of variables (cross-tables) can be generated in $R$ using the table command we have used in previous section to explore frequency distributions. For example, if you want cross-tabulate sex and affection status in the data frame assoc, you can use
```
> table(sex, aff)
    aff
sex 0 1
    0 95 24
    19531
```

Here, the first variable (sex) is presented in rows and the second (affection status) in columns.

As is usually the case with R , the output may be saved as a new object (of class 'table', which is a variety of a matrix):

```
> a <- table(sex, aff)
> class(a)
```

[1] "table"
$>a$
aff
sex $0 \quad 1$
09524
19531
and this object may be analysed further.

For example, we can easily get the number of affected male with

```
> a[2, 2]
```

[1] 31
Alternatively, we can analyse the resulting contingency table a with more complex functions. If we want to see proportions in this table, we can use

```
> prop.table(a)
    aff
sex 0 1
    0 0.38775510 0.09795918
    1 0.38775510 0.12653061
```

Needless to say, this is equivalent to

```
> prop.table(table(assoc$sex, assoc$aff))
    0 1
    0 0.38775510 0.09795918
    10.38775510 0.12653061
```

In the above table, we see what proportion of people belong to four different classes (affected male, affected female, unaffected male and unaffected female). We may be though interested in the proportion of males in affected and unaffected. This may be achieved by

```
> prop.table(a, 2)
    aff
sex 0 1
    0 0.5000000 0.4363636
    10.5000000 0.5636364
```

saying us that $56.4 \%$ of affected are male.
Alternatively, we may be interested in proportion of affected among males/females. To answer this question, run

```
> prop.table(a, 1)
    aff
sex 0 1
    0 0.7983193 0.2016807
    1 0.7539683 0.2460317
```

saying us that $56.4 \%$ of male are affected.
Other useful contingency table analysis function is fisher.test, which implements the Fisher Exact Test of independence:

```
fisher.test(a)
```


## Fisher's Exact Test for Count Data

data: a
$p$-value $=0.4457$
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
0.6767592 .482869
sample estimates:
odds ratio
1.290313

Exploration of genetic data within base R, though possible, may be a bit of a pain. For example, we can easily generate contingency table of SNP5 vs affected status:

```
a <- table(aff, snp5)
> a
```

```
    snp5
aff A/A B/A B/B
    0
    1
```

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We can also look up what is the proportion of affected among different genotypic groups

```
> prop.table(a, 2)
    snp5
aff A/A B/A B/B
    0 0.7948718 0.7719298 0.8000000
    10.20512820.2280702 0.2000000
```

showing that proportion of cases is similar in 'A/A' and 'A/B' genotypic groups and somewhat decreased in ' $B / B$ '. It is easy to test if this affection is statistically independent of genotype by

```
> chisq.test(a)
    Pearson's Chi-squared test
data: a
X-squared = 0.2511, df = 2, p-value = 0.882
```

which gives (insignificant) genotypic association test on two degrees of freedom.
However, testing Hardy-Weinberg equilibrium, testing allelic effects, and even computation of allelic frequency is not so straightforward. Such specific genetic tests are implemented in special $R$ libraries, such as genetics and GenABEL and will be covered in later sections of this document.

At this moment we will switch to exploratory analysis of quantitative traits. We will make use of the srdta data supplied with GenABEL. As you can remember from an exercise, the library is loaded with library (GenABEL) and the data are loaded with data(srdta): Then the phenotypic data frame may be accessed through srdta@phdata.

## Exercise 5.

Explore srdta@phdata. How many observations and variables are presented in the data frame? What are the classes of these variables?

As it was mentioned before, the function summary() generates a summary statistics for an object. For example, to see summary for trait qt1, we can use
> summary(srdta@phdata\$qt1)

| Min. 1st Qu. | Median | Mean | 3rd Qu. | Max. | NA's |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -4.6000 | -0.9500 | -0.3100 | -0.2981 | 0.3800 | 3.2000 |

summary is quite useful function which may operate in different ways for objects of different classes. Try summary (srdta@phdata).

Histogram of srdta@phdata\$qt1


Figure 2.1: Histogram of qt1

With R , it is also easy to explore the data graphically. For example, the histogram for qt1 may be generated by

```
> hist(srdta@phdata$qt1)
```

(resulting histogram is shown at figure 2.1)
In similar manner, scatter-plots may be generated. To see relation between qt1 and qt3, you can run

```
> plot(srdta@phdata$qt1, srdta@phdata$qt3)
```

(resulting plot is shown at figure 2.2)
The mean, median, minimum and maximum of the distribution of the trait may be found out using functions mean, median, min and max, respectively. The variance and standard deviation can be computed with var and sd.

To compute correlation between two variables (or all variables in a matrix/data frame), use cor.

In GenABEL, there is a special function designed to facilitate phenotypic quality control. This function takes names of variables and a data frame as an input, and returns summary statistics, list of outliers (using False Discovery Rate) and graphs.

For example, to do QC of sex, age and qt3, try

```
> check.trait(c("sex", "age", "qt3"), srdta@phdata)
```



Figure 2.2: Scatter-plot of qt1 against qt3


Figure 2.3: Quality control graph for sex, age, qt3

```
Trait sex has 2500 measurements
Missing: 0 ( 0 %)
Mean = 0.51 ; s.d. = 0.5
NO outliers discovered for trait sex
Trait age has 2500 measurements
Missing: 0 ( 0 %)
Mean = 50.0378 ; s.d. = 7.060125
NO outliers discovered for trait age
Trait qt3 has 2489 measurements
Missing: 11 ( 0.44 %)
Mean = 2.60859 ; s.d. = 1.101154
NO outliers discovered for trait qt3
```

The corresponding graph is depicted at figure ??.

Before you start with the exercise: if a function returns unexpected results, and you are confident that syntax was right, checking help page is always a good idea!

## Exercise 6.

Explore phdata slot of srdta

1. How many people has age over 65 years?
2. What is the sex distribution (proportion of males) in the people over 65 years old?

## Exercise 7.

Explore variables in phdata slot of srdta

1. What is the mean, median, minimum and maximum age in the sample?
2. Compare the distribution of qt 3 in people younger and older than 65 years. Use function $\operatorname{sd}(A)$ to get standard deviation of $A$.
3. Produce distributions of different traits. Do you see something special?
4. What is correlation between qt3 and age?

### 2.4 Regression analysis

While contingency tables, bi-plots and correlation are powerful tools to analyse relations between pairs of variable, a more general framework allowing investigation of relation of an outcome to multiple predictors is regression. In R, function lm implements linear regression modelling, and function glm implements generalised linear regression. In this section, we will use these two functions to analyse quantitative an binary outcomes.

You can do linear regression to check if trait qt2 has relation with sex and age by

```
> a <- lm(srdta@phdata$qt2 ~ srdta@phdata$age + srdta@phdata$sex)
```

The results of analysis are stored in object 'a', which has class 'lm' and contains may sub-objects:

```
> class(a)
```

[1] "lm"

```
> names(a)
```

```
[1] "coefficients" "residuals" "effects" "rank"
[5] "fitted.values" "assign" "qr" "df.residual"
[9] "xlevels" "call" "terms" "model"
```

At this moment you do not need to understand all these sub-objects; the meaningful summary of analysis is produced with

```
> summary(a)
Call:
lm(formula = srdta@phdata$qt2 ~ srdta@phdata$age + srdta@phdata$sex)
Residuals:
    Min 1Q Median 3Q Max
-5.6498 -1.7953 -1.0328 -0.3148 883.0808
```

Coefficients:

|  | Estimate Std. Error t value $\operatorname{Pr}(>\|t\|)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (Intercept) | -1.55892 | 4.41667 | -0.353 | 0.724 |
| srdta@phdata\$age | 0.14022 | 0.08668 | 1.618 | 0.106 |
| srdta@phdata\$sex | 1.30377 | 1.22393 | 1.065 | 0.287 |
| Residual standard error: 30.59 on 2497 degrees of freedom |  |  |  |  |
| Multiple R-squared: 0.001518, Adjusted R-squar |  |  |  |  |
| F-statistic: 1.898 on 2 and 2497 DF, p-value: 0.1501 |  |  |  |  |

You can see that qt2 is not associated with age or sex.
As before, to make easy access to your data (basically, to avoid typing srdta@phdata before every trait name, you may attach the data to the search path:

```
> attach(srdta@phdata)
```

Then,the above expression to run linear regression analysis simplifies to:

```
> summary(lm(qt2 ~ age + sex))
Call:
lm(formula = qt2 ~ age + sex)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & 3Q & Max \\
-5.6498 & -1.7953 & -1.0328 & -0.3148 & 883.0808
\end{tabular}
```

Coefficients:
Estimate Std. Error t value $\operatorname{Pr}(>|\mathrm{t}|)$

| (Intercept) | -1.55892 | 4.41667 | -0.353 | 0.724 |
| :--- | ---: | ---: | ---: | ---: |
| age | 0.14022 | 0.08668 | 1.618 | 0.106 |
| sex | 1.30377 | 1.22393 | 1.065 | 0.287 |

Residual standard error: 30.59 on 2497 degrees of freedom
Multiple R-squared: 0.001518, Adjusted R-squared: 0.0007181
F-statistic: 1.898 on 2 and 2497 DF, p-value: 0.1501
with the same results.
Analysis of binary outcomes may be performed using glm function, using binomial family for the error distribution and the link function. For example, to figure out if your binary trait (bt) is associated with sex and age, you need to tell that this is binary trait:

```
> a <- glm(bt ~ age + sex, family = "binomial")
> summary(a)
Call:
glm(formula = bt ~ age + sex, family = "binomial")
Deviance Residuals:
    Min 1Q Median 3Q Max
```

$\begin{array}{lllll}-1.992 & -1.091 & -0.444 & 1.094 & 1.917\end{array}$

Coefficients:

```
    Estimate Std. Error z value Pr(>|z|)
```

(Intercept) -4.639958 $0.330519-14.038<2 e-16 * * *$
age $\quad 0.088860 \quad 0.006463 \quad 13.749<2 e-16 * * *$
sex $\quad 0.379593 \quad 0.084138 \quad 4.512 \quad 6.44 \mathrm{e}-06$ ***
--

(Dispersion parameter for binomial family taken to be 1)
Null deviance: 3450.5 on 2488 degrees of freedom
Residual deviance: 3216.5 on 2486 degrees of freedom
(11 observations deleted due to missingness)
AIC: 3222.5

Number of Fisher Scoring iterations: 4
There is strong association between bt and sex and age. If you want to characterise the strength of association to a binary trait with Odds Ratios, take the exponents of the regression coefficient. For example, the odds ratio associated with male is
$>\exp (0.3796)$
[1] 1.461700

## Chapter 3

## Introduction to genetic association analysis in $\mathbf{R}$


#### Abstract

When analyzing several (dozens of) SNPs, facilities of base R are sufficient and efficient for data storage and analysis. Few specific test, such as these of Hardy-Weinberg Equilibrium (HWE) and Linkage Disequilibrium (LD), are implemented in different libraries, e.g. genetics and GenABEL .

In this section, we will describe library genetics and will make use of it to guide you through simple genetic analysis exercise using a small example data set. In the last part, you will investigate a bigger data set as based on the knowledge obtained in the first part, and will answer the questions.


### 3.1 Characterisation of genetic data

### 3.2 Exploring genetic data with library genetics

Library genetics was written by Gregory R . Warnes to facilitate analysis of genetic data in R . This library

- Implements genetic analysis tests, such as test for Hardy-Weinberg equilibrium and Linkage disequilibrium.
- Implements new data classes, such as genotype, haplotype and LD.data.frame.
- Modifies default R functions, such as summary and plot to correctly analyse and present these new classes.
- Facilitates export of the data from R to the formats supported by such genetic analysis packages as GenePop and QTDT.

Start $R$ by double-click on the file ge03d1p1.RData. Load library genetics, which we will need for testing HWE and computations of LD by

```
> library(genetics)
```

The file you have loaded contains single data frame assocg. Let us briefly explore it:

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```
> class(assocg)
[1] "data.frame"
> names(assocg)
[1] "subj" "sex" "aff" "qt" "snp4" "snp5" "snp6"
> dim(assocg)
[1] 250 7
```

You can see that assocg looks remarkably similar to the previously explored data frame assoc (section 2.2, page 20). Indeed, they are almost equivalent. Let us present the data for the subjects 5 to 15 and compare this output to that presented on page 22 .

|  | subj | sex | aff qt | snp4 | snp5 | snp6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1533 | 1533 | 0 | 00.1009220 | A/B | B/A | B/A |
| 2466 | 2466 | 1 | $0-0.1724321$ | A/B | A/A | A/A |
| 2425 | 2425 | 0 | $0-0.3378473$ | B/B | A/A | A/A |
| 1068 | 1068 | 0 | $0-1.7112925$ | A/A | B/B | <NA> |
| 198 | 198 | 1 | $0-0.4815822$ | A/B | B/A | B/A |
| 1496 | 1496 | 1 | 1.2281232 | A/A | B/B | B/B |
| 909 | 909 | 0 | 0.5993945 | A/B | B/A | B/A |
| 1213 | 1213 | 0 | 01.9792190 | A/A | B/B | B/B |
| 181 | 181 | 1 | 01.5435921 | A/A | B/B | B/B |
| 1783 | 1783 | 0 | $0-1.6242738$ | A/B | B/A | B/A |
| 1914 | 1914 | 0 | $0-0.5160331$ | A/A | B/B | B/B |

The data are identical. However, the SNP data presented in the new data frame have special class genotype, as implemented in genetics library:

```
> class(assocg$snp4)
```

```
[1] "genotype" "factor"
```

Previously, the SNP genotypes were coded as characters. This new way of presentation allows library genetics to recognise the SNP data as genetic and analyse them accordingly.

Let us attach the assocg data frame and explore what data analysis advantages are achieved by application of library genetics.
> attach (assocg)
As we noted in section 2.2 testing Hardy-Weinberg equilibrium, testing allelic effects, and even computation of allelic frequency is not so straightforward in base R. These tests, are, however, easy with library genetics. To see the allelic frequencies and other summary statistics for a SNP, you can use

```
> summary(snp4)
```

```
Number of samples typed: 243 (97.2%)
Allele Frequency: (2 alleles)
    Count Proportion
A 323 0.66
B 163 0.34
NA 14 NA
Genotype Frequency:
        Count Proportion
B/B 29 0.12
A/B 105 0.43
A/A 109 0.45
NA 7 NA
Heterozygosity (Hu) = 0.4467269
Poly. Inf. Content = 0.3464355
```

To check these characteristics in controls and cases separately, you can use

```
> summary(snp4[aff == 0])
```

Number of samples typed: 190 (97.9\%)
Allele Frequency: (2 alleles)
Count Proportion
$\begin{array}{lll}\text { A } & 255 & 0.67\end{array}$
$\begin{array}{ll}\text { B } & 125 \quad 0.33\end{array}$
NA 8 NA

| Genotype Frequency: |  |  |
| :---: | :---: | :---: |
| Count Proportion |  |  |
| B/B | 22 | 0.12 |
| A/B | 81 | 0.43 |
| A/A | 87 | 0.46 |
| NA | 4 | NA |

Heterozygosity (Hu) = 0.4426469
Poly. Inf. Content $=0.3440288$
> summary (snp4[aff == 1])
Number of samples typed: 53 ( $94.6 \%$ )
Allele Frequency: (2 alleles)
Count Proportion
$\begin{array}{lll}\text { A } & 68 & 0.64\end{array}$
$\begin{array}{ll}\text { B } & 38 \\ \text { B }\end{array}$
NA 6 NA

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```
Genotype Frequency:
    Count Proportion
B/B 
A/B 24 0.45
A/A 22 0.42
NA 3 NA
Heterozygosity (Hu) = 0.4643306
Poly. Inf. Content = 0.3541731
```

Let us check if HWE holds for the SNPs described in this data frame. We can do exact test for HWE by

```
> HWE.exact(snp4)
```

Exact Test for Hardy-Weinberg Equilibrium
data: snp4
N11 = 109, N12 = 105, N22 = 29, N1 = 323, N2 = 163, p-value $=0.666$

If you want to check HWE using controls only, you can do it by

```
> HWE.exact(snp4[aff == 0])
```

Exact Test for Hardy-Weinberg Equilibrium

```
data: snp4[aff == 0]
N11 = 87, N12 = 81, N22 = 22, N1 = 255, N2 = 125, p-value = 0.6244
```

Let us check if the there is LD between snp4 and snp5:

```
> LD(snp4, snp5)
```

Pairwise LD

|  | D | D' | Corr |
| :---: | :---: | :---: | :---: |
| Estimates: | 0.2009042 | 0.9997352 | 0.8683117 |
|  | X^2 P- | value N |  |
| LD Test: 3 | 4.3636 | 0235 |  |

The output shows results of the test for significance of LD, and estimates of the magnitude of LD ( $D^{\prime}$ and correlation, $r$ ). To obtain $r^{2}$, you can either square the correlation manually
> $0.8683117 * 0.8683117$
[1] 0.7539652
or simply ask LD() to report it by

```
LD(snp4, snp5)$"R^2"
```

[1] 0.7539652

The latter command is possible because the LD() function actually computes more things than it reports. This is quite common for $R$ functions. You can apply names () function to the analysis objects to see (at least part of) what was actually computed. Try
> ld45 <- LD (snp4, snp5)
and check what are the sub-objects contained in this analysis object
> names(ld45)
[1] "call" "D" "D'" "r" "R^2" "n" "X^2"
[8] "P-value"
Any of these variables can be accessed through object\$var syntax, e.g. to check $D^{\prime}$ we can use
> ld45\$"D'"
[1] 0.9997352

To check LD for more that two SNPs, we can compute an LD analysis object by

```
> ldall <- LD(data.frame(snp4, snp5, snp6))
```

and later check

```
> ldall$"P-value"
\begin{tabular}{lrrr} 
& snp4 & snp5 & snp6 \\
snp4 & NA & 0 & 0 \\
snp5 & NA & NA & 0 \\
snp6 & NA & NA & NA
\end{tabular}
```

to see significance,
> ldall\$"D'"

|  | snp4 | snp5 | snp6 |
| :--- | ---: | ---: | ---: |
| snp4 | NA | 0.9997352 | 0.8039577 |
| snp5 | NA | NA | 0.9997231 |
| snp6 | NA | NA | NA |

for $D^{\prime}$ and
> ldall\$"R^2"
snp4 snp5 snp6
snp4 NA 0.75396520 .5886602
snp5 NA NA 0.8278328
snp6 NA NA NA
for $r^{2}$.
You can also present e.g. $r^{2}$ matrix as a plot by


Figure 3.1: $r^{2}$ plot for snp4, snp5 and snp6

```
> image(ldall$"R^2")
```

A more neat way to present it requires specification of the set of threshold (break points) and colors to be used (you do not need to try this example if you do not want):

```
> image(ldall$"R^2", breaks = c(0.5, 0.6, 0.7, 0.8, 0.9, 1), col = heat.colors(5))
```

Resulting plot is shown at figure 3.1 .

For any $R$ command, you can get help by typing help(command). Try help(image) if you are interested to understand what are "breaks" and "col"; or try help(heat.colors) to figure this color schema out.

Similar to our HWE checks, we may want to compute (and compare) LD in cases and controls separately:

```
> ldcases <- LD(data.frame(snp4, snp5, snp6)[aff == 1, ])
> ldcases$"R`2"
    snp4 snp5 snp6
snp4 NA 0.7615923 0.6891558
snp5 NA NA 0.8943495
snp6 NA NA NA
```



Figure 3.2: $r^{2}$ plot for snp4, snp5 and snp6. Above diagonal: LD in cases; below: controls

```
> ldcontr <- LD(data.frame(snp4, snp5, snp6)[aff == 0, ])
> ldcontr$"R`2"
    snp4 snp5 snp6
snp4 NA 0.7512458 0.5616395
snp5 NA NA 0.8075894
snp6 NA NA NA
```

and even present it results for cases and controls on the same graph (you do not need to produce this graph, which is presented at the figure 3.2):

```
> image(ldcases$"R^2", breaks = c(0.5, 0.6, 0.7, 0.8, 0.9, 1),
+ col = heat.colors(5))
> image(t(ldcontr$"R^2"), breaks = c(0.5, 0.6, 0.7, 0.8, 0.9, 1),
+ col = heat.colors(5), add = T)
```


### 3.3 Genetic association analysis

### 3.4 Example association analysis

Now, after we have described genetic and phenotypic data separately, we are ready to test association between these two. In previous sections, we showed

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that association between a binary trait and genotype may be analysed using contingency tables (functions table, prop.table, fisher.test, etc.). The association between a quantitative trait and genotype may be done using correlations, T-test, etc.

However, a more flexible analysis is possible when using regression modelling. First, we will investigate relation between the quantitative trait qt and the SNPs by using linear regression

```
> mg <- lm(qt ~ snp4)
```

The lm command fits linear regression model to the data and returns an analysis object. The summary of analysis may be generated with

```
> summary(mg)
Call:
lm(formula = qt ~ snp4)
Residuals:
\begin{tabular}{rrrrr} 
Min & 1Q & Median & 3Q & Max \\
-2.63700 & -0.62291 & -0.01225 & 0.58922 & 3.05561
\end{tabular}
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.081114 0.092517 -0.877 0.382
snp4A/B -0.108366 0.132079 -0.820 0.413
snp4B/B 
Residual standard error: 0.9659 on 240 degrees of freedom
    (7 observations deleted due to missingness)
Multiple R-squared: 0.003049, Adjusted R-squared: -0.005259
F-statistic: 0.367 on 2 and 240 DF, p-value: 0.6932
```

From the summary output, it is clear that the model assumes arbitrary (estimated) effects of the genotypes $\mathrm{AA}, \mathrm{AB}$ and BB . Neither effect of AB nor BB is significant in this case. The global test on two degrees of freedom (bottom of the output) is also not significant.

If you want to include some covariate into your model, e.g. sex, you can easily do that by adding the term to the formula:

```
> summary(lm(qt ~ sex + snp4))
Call:
lm(formula = qt ~ sex + snp4)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & 3Q & Max \\
-2.664422 & -0.624169 & -0.008752 & 0.597045 & 3.080857
\end{tabular}
Coefficients:
    Estimate Std. Error t value Pr}(>|t|
(Intercept) -0.110298 0.115260 -0.957 0.340
```

```
sex }\quad0.053018 0.124493 0.426 0.671 
snp4A/B 
snp4B/B -0.002452 0.202340
Residual standard error: 0.9676 on 239 degrees of freedom
    (7 observations deleted due to missingness)
Multiple R-squared: 0.003805, Adjusted R-squared: -0.0087
F-statistic: 0.3043 on 3 and 239 DF, p-value: 0.8223
```

You can also allow for interaction by using the "*" operator

```
> summary(lm(qt ~ sex * snp4))
Call:
lm(formula = qt ~ sex * snp4)
Residuals:
\begin{tabular}{rrrrr} 
Min & 1Q & Median & 3Q & Max \\
-2.570485 & -0.645961 & -0.002641 & 0.610938 & 3.019696
\end{tabular}
Coefficients:
    Estimate Std. Error t value Pr (>|t|)
(Intercept) -0.20579 0.13834 -1.487 0.138
sex 0.22649 0.18647 1.215 0.226
snp4A/B 0.05222 0.19024 0.274 0.784
snp4B/B 0.18071 0.28576 0.632 0.528
sex:snp4A/B -0.30191 0.26566 -1.136 0.257
sex:snp4B/B -0.35508 0.40531 -0.876 0.382
Residual standard error: 0.9684 on 237 degrees of freedom
    (7 observations deleted due to missingness)
Multiple R-squared: 0.01041, Adjusted R-squared: -0.01047
F-statistic: 0.4984 on 5 and 237 DF, p-value: 0.7773
```

Note that both main effects of sex and snp4, and also effects of interaction are estimated in this model.

Of interest in genetic studies may be three other models: additive, dominant and recessive.

The additive model assumes that the difference between mean trait's values between ' AA ' and ' BB ' is twice the difference between ' AA ' and ' BB ', that is the mean value of the trait for heterozygous genotypes is right in between the two homozygotes. To test additive model, we first need to recode the predictor (genotype) as a numeric factor to be used as covariate. This can be easy done with function as.numeric:

```
> add4 <- as.numeric(snp4) - 1
```

We can check if recoding was done correctly by producing the table

```
> table(snp4, add4)
```


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```
        add4
snp4 0 1 2
    A/A 109 0 0
    A/B 0 105 0
    B/B 0 0 29
```

Now to test the additive model run

```
> summary(lm(qt ~ add4))
```

Call:
$\operatorname{lm}\left(\right.$ formula $=q t{ }^{\sim}$ add4)
Residuals:

| Min | 1Q | Median | 3Q | Max |
| ---: | ---: | ---: | ---: | ---: |
| -2.54813 | -0.62104 | -0.02754 | 0.60584 | 3.00652 |

Coefficients:
Estimate Std. Error t value $\operatorname{Pr}(>|t|)$

| (Intercept) -0.10476 | 0.08710 | -1.203 | 0.230 |
| :--- | :--- | :--- | :--- |


| add4 | -0.03563 | 0.09133 | -0.390 | 0.697 |
| :--- | :--- | :--- | :--- | :--- |

Residual standard error: 0.9651 on 241 degrees of freedom
(7 observations deleted due to missingness)
Multiple R-squared: 0.0006313, Adjusted R-squared: -0.003516
F-statistic: 0.1522 on 1 and 241 DF, p-value: 0.6968

The model assuming dominant action of the 'A' allele means that the means of genotypes ' AA ' and ' AB ' are the same. This is equivalent to the model of recessive action of 'B' allele. To code SNP4 according to this model, we can use function replace:

```
> dom4 <- add4
> dom4[dom4 == 2] <- 1
> table(snp4, dom4)
        dom4
snp4 0 1
    A/A 109 0
    A/B 0 105
    B/B O 29
```

To test association with a binary outcome, we will use function glm with binomial family:

```
> summary(glm(aff ~ snp4, family = "binomial"))
Call:
glm(formula = aff ~ snp4, family = "binomial")
```

Deviance Residuals:
$\begin{array}{lllll}\text { Min } & \text { 1Q Median } & \text { 3Q }\end{array}$

| -0.7433 | -0.7204 | -0.6715 | -0.6715 | 1.7890 |
| :--- | :--- | :--- | :--- | :--- |

Coefficients:
Estimate Std. Error z value $\operatorname{Pr}(>|z|)$

| (Intercept) | -1.3749 | 0.2386 | -5.761 | $8.35 \mathrm{e}-09$ |
| :--- | ---: | ---: | :---: | :---: | *** $^{\text {(In }}$

---

(Dispersion parameter for binomial family taken to be 1)
Null deviance: 254.91 on 242 degrees of freedom
Residual deviance: 254.58 on 240 degrees of freedom
(7 observations deleted due to missingness)
AIC: 260.58
Number of Fisher Scoring iterations: 4
To make a test of global significance of the SNP effect, you can use

```
> anova(glm(aff ~ snp4, family = "binomial"), test = "Chisq")
Analysis of Deviance Table
Model: binomial, link: logit
Response: aff
Terms added sequentially (first to last)
    Df Deviance Resid. Df Resid. Dev P(>|Chi|)
NULL 242 254.91
snp4 2 0.32894 240 254.58 0.8483
```

In the manner similar to that described for quantitative traits, additive and dominance/recessive models can be tested by proper coding of the genotypic variable, e.g. to test the additive model, use

```
> summary(glm(aff ~ as.numeric(snp4), family = "binomial"))
Call:
glm(formula = aff ~ as.numeric(snp4), family = "binomial")
Deviance Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & 3Q & Max \\
-0.7548 & -0.7139 & -0.6747 & -0.6747 & 1.7842
\end{tabular}
Coefficients:
    Estimate Std. Error z value Pr(>|z|)
```


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```
(Intercept) -1.4913 0.4164 -3.581 0.000342 ***
as.numeric(snp4) 0.1272 0.2268 0.561 0.574994
---
```



```
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 254.91 on 242 degrees of freedom
Residual deviance: 254.60 on 241 degrees of freedom
    (7 observations deleted due to missingness)
AIC: 258.60
Number of Fisher Scoring iterations: 4
```

Now you have learned all commands necessary to answer the questions of the next section.

Exit R by typing $q$ () command (do not save image) and and proceed to the self exercise.

### 3.5 Exercise

Start R by double-click over the file ge03d1p2.RData. Explore the data frame present and answer the questions.

Exercise 1 How many SNPs are described in this data frame?
Exercise 2 What is the frequency (proportion) of snp1 allele A? What is its frequency in these affected (aff==1)?

Exercise 3 How many cases and controls are present?
Exercise 4 If all subjects are used to test HWE, are there any SNPs out of $H W E$ at nominal $P \leq 0.05$ ? Which ones?

Exercise 5 If only controls are used to test the SNPs which are out of HWE in total sample, are these still out of HWE?

Exercise 6 Which SNP pairs are in strong LD ( $r^{2} \geq 0.8$ )?
Exercise 7 For SNPs in strong LD, what is $r^{2}$ for separate samples of cases and controls?

Exercise 8 Is there significant association between affection status and sex? What is $P$-value for association?

Exercise 9 Is association between the disease and qt significant?
Exercise 10 Which SNPs are associated with the quantitative trait qt at nominal $P \leq 0.05$ ? Use 2 d.f. test.

Exercise 11 Test each SNP for association with the affection status, using 2 d.f. test. Which SNPs are significantly associated at nominal $P \leq 0.05$ ? How can you describe the model of action of the significant SNPs?

Exercise 12 For the SNPs selected in previous question, test association using additive model. Which SNPs are still associated?

Exercise 13 If you adjust the analysis under additive model (question 12) for significant covariates which you discovered in questions 8 and 9 , are these findings still significant?

Exercise 14 Test association between aff and snp5 and snp10, allowing for the SNPs interaction effect. Use arbitrary (not an additive) model. Do you observe significant interaction? How can you describe the model of concert action of snp5 and snp10?

## Chapter 4

## Introduction to GenABEL


#### Abstract

In this section, you will become familiar with the GenABEL library, designed for GWA analysis. Compared to genetics package, it provides specific facilities for storage and manipulation of large amounts of data, very fast tests for GWA analysis, and special functions to analyse and graphically present the results of GWA analysis (thus "analysis of analysis").

Start R and load GenABEL library using command


```
> library(GenABEL)
```

After that, load the data with the command

```
> data(srdta)
```


### 4.1 General description of gwaa.data-class

The object you have loaded, srdta, belongs to the gwaa.data class. This is a special class developed to facilitate GWA analysis.

In GWA analysis, different types of data are used. These include the phenotypic and genotypic data on the study participants and chromosome and location of every SNP. For every SNP, it is desirable to know the details of coding (what are alleles? - A, T, G, C? - and what is the strand - '+' or '-', 'top' or 'bot'? - this coding is for).

One could attempt to store all phenotypes and genotypes together in a single table, using, e.g. one row per study subject; than the columns will correspond to study phenotypes and SNPs. For a typical GWA data set, this would lead to a table of few thousands rows and few hundreds of thousands of columns. Such a format is generated when one downloads HapMap data for a region. To store GWA data in such tables internally, within R, proves to be inefficient. In GenABEL, special data class, gwaa.data-class is used to store GWA data. The structure of this data class is shown at the figure 4.1.

An object of some class has "slots" which may contain actual data or objects of other classes. The information stored at a particular slot of an object can be accessed by command object@slot.

At the first level, a gwaa.data-class object has slot phdata, which contains all phenotypic information in a data frame (data.frame-class object). The


Figure 4.1: Structure of gwaa.data-class. In every box, first line contains the object and slot names, second line describes the class of this object, and third line describes what information is contained.
rows of this data frame correspond to study subjects, and the columns correspond to the variables. There are two default variables, which are always present in phdata. The first of these is "id", which contains study subject identification code. This identification code can be arbitrary character, but every person must be coded with an unique ID. The second default variable is "sex", where males are coded with ones ("1") and females are coded with zero (" 0 "). It is important to understand that this data frame is not supposed to be directly modified by the user. In particular, it is extremely important to remember that one should not directly add subjects to the table, change the values of "id" and "sex", and change the order of subjects in phdata unless this one is really understands the way GenABEL works. One also should not run such data manipulation functions as merge, cbind and rbind - exactly because they may change the number of subjects or interfere with the order. On the other hand, it is OK to add more variables to the data frame through direct computations, for example, if one wishes to add computed body mass index, it is OK to run the command like
obj@phdata\$bmi <- obj@phdata\$weight/((obj@phdata\$height) 2$)$
To add many variables to phdata, special GenABEL function add. phdata should be used.

The other slot of an object of gwaa.data-class is slot gtdata, which contains all GWA genetic information in an object of class snp. data class (figure 4.1). This class, in turn, has slots nids, containing the number of study subjects, idnames, containing all ID names of these subjects, nsnps, containing the number of SNPs typed, snpnames, containing the SNP names, chromosome, containing the name of the chromosome the SNPs belong to and slot map with map position of SNPs, and slot male, containing the sex code for the subjects ( $1=$ male, $0=$ female). The latter is identical to the "sex" variable contained in the phdata, but is duplicated here because many operations with purely genetic data, in particular these concerning analysis of sex chromosomes, depend on the sex. The strand information is presented in the slot strand. GenABEL codes strand as "+" (forward), "-" (reverse) or "u" (unknown). Of cause, if you prefer top/bottom coding, this information may be stored in the same form - you will just need to remember that " + " corresponds to e.g. "top", and "-" to "bottom" strand. The allelic coding is presented in slot coding. Coding for every allele is presented with a pair of characters, for example "AG". Thus, for such polymorphism, you may expect "AA", "AG" and "GG" genotypes to be found in population. The order (that is "AG" vs "GA") is important - the first allele reported is the one which will be used as a reference in association analysis, and thus the effects are reported for the second allele. To avoid memory overheads, the strand and coding information is internally stored as snp. strand-class and snp.coding-class. Information can be converted to human-readable format using as.character function.

If, for example, you would like to know, how many SNPs were included in the study (slot nsnps of the slot gtdata of srdta), you need to run command

```
> srdta@gtdata@nsnps
```

[1] 833
Thus, 833 SNPs were typed in the study. You can access information stored in any slot in this manner.

You may want to read the general GenABEL man page using help(GenABEL). To see help on gwaa.data-class, you can use help("gwaa.data-class") (mind the quotation marks!).

## Summary:

- An object of some class has "slots" which may contain actual data or objects of other classes. The information stored at a particular slot of an object can be accessed by command object@slot.
- GenABEL uses special data class, gwaa.data-class, to store GWA data.


## Exercise 8.

Explore srdta.

1. How many people are included in the study?
2. How many of these are males?
3. How many are females?
4. What is male proportion?

## Exercise 9.

Explore slot containing map (map) and slot containing SNP names (snpnames) of the gtdata slot of srdta.

1. What are names of markers located after $2,490,000$ b.p.?
2. Between $1,100,000$ and $1,105,000$ b.p.?

### 4.2 Sub-setting and coercing gwaa.data

It is possible to sub-set the object, which stores the GWA data in the manner similar to the described above. Very primitively, you may think of an object of class gwaa.data as a matrix whose rows correspond to study subjects and columns correspond to SNPs studied (though the actual object is a way more complicated). For example, if we would like to investigate what is the content of srdta for the first 5 people and 3 SNPs, we can run

```
> ssubs <- srdta[1:5, 1:3]
> class(ssubs)
```

```
[1] "gwaa.data"
attr(,"package")
[1] "GenABEL"
```

ssubs

|  | id sex | age | qt1 | qt2 | qt3 | bt |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | p1 | 1 | 43.4 | -0.58 | 4.46 | 1.43 |
| 2 | p2 | 1 | 48.2 | 0.80 | 6.32 | 3.90 |
| 3 | p3 | 0 | 37.9 | -0.52 | 3.26 | 5.05 |

```
4 p4 1 53.8 -1.55 888.00 3.76 1
5 p5 1 47.5 0.25 5.70 2.89 1
@nids = 5
@nsnps = 3
@nbytes = 2
@idnames = p1 p2 p3 p4 p5
@snpnames = rs10 rs18 rs29
@chromosome = 1 1 1
@coding = 08 0b 0c
@strand = 01 01 02
@map = 2500 3500 5750
@male = 1 1 0 1 1
@gtps =
404040
4040 00
```

As you can see, by sub-setting we obtained a smaller object of gwaa.dataclass, with all its slots. Most of the information is straightforward and does not need further explanation.

There are though three slots which are not human-readable, i.e. @coding, @strand, and @gtps. These are coded using new classes based in R raw data type; these can be converted in human-readable format using a variety of 'as.*' functions. For example, to see human-readable information on coding and strand, let us try

```
> as.character(ssubs@gtdata@coding)
rs10 rs18 rs29
"TG" "GA" "GT"
```

This tells what alleles are observed in the three SNPs, and which alleles will be used as a reference in association analysis.

To see the strand, use

```
> as.character(ssubs@gtdata@strand)
rs10 rs18 rs29
    "+" "+" "-"
```

The slot gtps contains the SNP data, and is not readable, because the information is compressed (each element contains data on up to four genotypes). To get human-readable information, an object of class snp.data-class (e.g. srdta@gtdata) can be coerced to a character using the same as.character() function:

```
> as.character(ssubs@gtdata)
    rs10 rs18 rs29
p1 "T/T" "G/G" "G/G"
p2 "T/T" "G/G" NA
p3 "T/T" "G/G" NA
p4 "T/T" "G/G" NA
p5 "T/T" "G/A" "G/G"
```

For @gtps conversion to other formats are possible as well. Other useful coercion is to "numeric":

```
> as.numeric(ssubs@gtdata)
    rs10 rs18 rs29
p1 0 0 0
p2 0 0 NA
p3 0 0 NA
p4 0 0 NA
p5 0
```

You can see that conversion to numeric data type uses information on both underlying genotypes and coding. When coding is "GA", as is for the rs18 (the second SNP), homozygotes for the first allele, as specified by coding ("G") are converted to zeros ("0"), heterozygotes are converted to ones ("1"), and homozygotes for the second allele ("A") are converted to twos ("2"). Clearly, when numerically converted data are used for association analysis, the effects will be estimated for the second allele, while first will be used as a reference.

Genotypic data converted to standard R "numeric" format can be used in any further analysis. Homozygotes of one type are coded as "0", heterozygotes are coded as " 1 " and other type of homozygotes is coded as " 2 ". You can think of this as the number of allele of "B" type.

Several useful genetic analysis libraries were developed for R. These include genetics (analysis of linkage disequilibrium and many other useful functions) and haplo.stats (analysis of association between traits and haplotypes). These use there own genetic data formats.

One can translate GenABEL genetic data to the format used by "genetics" library by as.genotype():

```
> as.genotype(ssubs@gtdata)
    rs10 rs18 rs29
p1 T/T G/G G/G
p2 T/T G/G <NA>
p3 T/T G/G <NA>
p4 T/T G/G <NA>
p5 T/T G/A G/G
```

To translate GenABEL data to the format used by "haplo.stats" you can use function as.hsgeno()

```
> as.hsgeno(ssubs@gtdata)
    rs10.a1 rs10.a2 rs18.a1 rs18.a2 rs29.a1 rs29.a2
\begin{tabular}{rrrrrrr} 
p1 & 1 & 1 & 1 & 1 & 1 & 1 \\
p2 & 1 & 1 & 1 & 1 & NA & NA \\
p3 & 1 & 1 & 1 & 1 & NA & NA \\
p4 & 1 & 1 & 1 & 1 & NA & NA \\
p5 & 1 & 1 & 1 & 2 & 1 & 1
\end{tabular}
```

Actually, most users will not need the latter function, as GenABEL provides a functional interface to "haplo.stats" (such GenABEL functions as scan.haplo() and scan.haplo.2D()).

It is possible to select sub-sets of gwaa.data-class based not only on index (e.g. first 10 people and SNP number 33), but also based on names.

For example, if we would like to retrieve phenotypic data on people with IDs "p141", "p147" and "p2000", we can use

```
> srdta[c("p141", "p147", "p2000"), ]@phdata
    id sex age qt1 qt2 qt3 bt
141 p141 0 47.2 0.51 5.23 2.17 0
147 p147 0 43.2 0.14 4.47 1.73 0
2000 p2000 0 43.1 -1.53 2.78 2.70 1
```

here, the first part of expression sub-sets srdta on selected IDs, and the second tells which part of the retrieved sub-set we want to see. You can try srdta[c("p141", "p147", "p2000"),], but be prepared to see long output, as all information will be reported.

In similar manner, we can also select on SNP name. For example, if we are interested to see information on SNPs "rs10" and "rs29" for above people, we can run

```
> srdta[c("p141", "p147", "p2000"), c("rs10", "rs29")]
    id sex age qt1 qt2 qt3 bt
141 p141 0 47.2 0.51 5.23 2.17 0
147 p147 0 43.2 0.14 4.47 1.73 0
2000 p2000 0 43.1 -1.53 2.78 2.70 1
@nids = 3
@nsnps = 2
@nbytes = 1
@idnames = p141 p147 p2000
@snpnames = rs10 rs29
@chromosome = 1 1
@coding = 08 0c
@strand = 01 02
@map = 2500 5750
@male = 0 0 0
@gtps =
4040
```

To see the actual genotypes for the above three people and two SNPs, use

```
> as.character(srdta[c("p141", "p147", "p2000"), c("rs10", "rs29")])
        rs10 rs29
p141 "T/T" "G/G"
p147 "T/T" "G/G"
p2000 "T/G" "G/T"
```

```
> as.numeric(srdta[c("p141", "p147", "p2000"), c("rs10", "rs29")])
    rs10 rs29
p141 0 0
p147 0 0
p2000 1 1
```

Exercise 10.
Explore genotypes for SNP "rs114".

1. What is the coding and which allele is the reference one?
2. What is the frequency of non-reference ("effective") allele in total sample?

3 . What is the frequency of effective allele in male?
4. What is the frequency of effective allele in female?
5. What is the frequency of the reference allele in total sample, males and females?

## Summary:

- It is possible to obtain subsets of objects of gwaa.data-class and snp.data-class using standard 2D sub-setting model [i,j], where i corresponds to study subjects and $j$ corresponds to SNPs.
- It is possible to provide ID and SNP names instead of indexes when subsetting an object of class gwaa.data-class.
- Function as.numeric() converts genotypic data from snp.data-class to regular integer numbers, which can be used in analysis with R.
- Function as.character() converts genotypic data from snp.data-class to character format.
- Function as.genotype() converts genotypic data from snp.data-class to the format used by library genetics.
- Function as.hsgeno() converts genotypic data from snp.data-class to the format used by library haplo.stats.


### 4.3 Exploring genetic data

Implementation of function summary () to snp. data class is very useful in genetic data exploration and quality control (QC). Let us try application of this function to the ssubs:

```
> a <- summary(ssubs)
>a
$phdata
```

Length:5 Min. :0.0 Min. :37.90 Min. :-1.55
Class :character 1st Qu.:1.0 1st Qu.:43.40 1st Qu.:-0.58
Mode :character Median :1.0 Median :47.50 Median :-0.52
Mean :0.8 Mean :46.16 Mean :-0.32
3rd Qu.:1.0 3rd Qu.:48.20 3rd Qu.: 0.25
Max. :1.0 Max. :53.80 Max. : 0.80
qt2 qt3 bt
Min. : 3.26 Min. :1.430 Min. :0.0
1st Qu.: 4.46 1st Qu.:2.890 1st Qu.:1.0
Median : 5.70 Median :3.760 Median :1.0
Mean :181.55 Mean :3.406 Mean :0.8
3rd Qu.: 6.32 3rd Qu.:3.900 3rd Qu.:1.0
Max. :888.00 Max. :5.050 Max. :1.0

```
\$gtdata
    NoMeasured CallRate Q. 2 P. 11 P. 12 P. 22 Pexact Fmax Plrt
\(\begin{array}{llllllllll}\text { rs10 } & 5 & 1.0 & 0.0 & 5 & 0 & 0 & 1 & 0.0000000 & 1.0000000\end{array}\)
\(\begin{array}{lllllllllll}\text { rs18 } & 5 & 1.0 & 0.1 & 4 & 1 & 0 & 1 & -0.1111111 & 0.7386227\end{array}\)
\(\begin{array}{llllllllll}\text { rs29 } & 2 & 0.4 & 0.0 & 2 & 0 & 0 & 1 & 0.0000000 & 1.0000000\end{array}\)
    Chromosome
rs10 1
rs18 1
rs29 1

In the first section, the summary is generated for phenotypic data. In the second section, summary is generated for genotypic data. In this section, NoMeasured refers to the number of genotypes scores, CallRate to the proportion of these, Q. 2 is the frequency of the 'B' allele. The counts in three genotypic classes are provided next. Pexact refers to exact P-value for the test of Hardy-Weinberg equilibrium.

As you've seen above, an object of the class gwaa.data-class or snp. dataclass is sub-settable in standard manner: [ \(i, j]\), where \(i\) is an index of a study subject and \(j\) is an index of a SNP. Importantly, i could be a list of indexes:
```

> vec <- which(srdta@phdata\$age >= 65)
> vec

```
\begin{tabular}{lrrrrrrrrrrrrrrr} 
[1] & 64 & 122 & 186 & 206 & 207 & 286 & 385 & 386 & 492 & 514 & 525 & 536 & 545 & 565 & 613 \\
[16] & 632 & 649 & 673 & 701 & 779 & 799 & 981 & 1008 & 1131 & 1186 & 1223 & 1281 & 1383 & 1471 & 1489 \\
[31] & 1501 & 1565 & 1584 & 1673 & 1679 & 1782 & 1821 & 1832 & 1866 & 1891 & 1953 & 2081 & 2085 & 2140 & 2224 \\
[46] & 2268 & 2291 & 2384 & 2420 & 2453 & & & & & & & & & &
\end{tabular}
> summary(srdta@gtdata[vec, 1:3])
\begin{tabular}{lrrrrrrrr} 
& NoMeasured CallRate & Q.2 & P.11 & P. 12 & P. 22 & Pexact & Fmax \\
rs10 & 48 & 0.96 & 0.1354167 & 36 & 11 & 1 & 1.0000000 & 0.02131603 \\
rs18 & 47 & 0.94 & 0.2765957 & 25 & 18 & 4 & 0.7245853 & 0.04298643 \\
rs29 & 45 & 0.90 & 0.1555556 & 32 & 12 & 1 & 1.0000000 & -0.01503759
\end{tabular}
rs10 0.8843626
rs18 \(0.7697067 \quad 1\)
rs29 \(0.9188943 \quad 1\)

This shows summary of first three genotypes for people with age greater then or equal to 65 y.o. The same result may be achieved by sub-setting using a vector of logical values:
```

> vec <- (srdta@phdata\$age >= 65)
> table(vec)
vec
FALSE TRUE
2450 50
> summary(srdta@gtdata[vec, 1:3])
NoMeasured CallRate Q.2 P.11 P.12 P.22 Pexact Fmax
rs10 48 0.96 0.1354167 36 11 1 1.0000000 0.02131603
rs18 47 0.94 0.2765957 25 18 4 0.7245853 0.04298643
rs29 45 0.90 0.1555556 32 12 1 1.0000000-0.01503759
Plrt Chromosome
rs10 0.8843626 1
rs18 0.7697067 1
rs29 0.9188943 1

```
or a list with IDs of study subjects:
```

> vec1 <- srdta@gtdata@idnames[vec]
> vec1

```
```

    [1] "p64" "p122" "p186" "p206" "p207" "p286" "p385" "p386" "p492"
    [10] "p514" "p525" "p536" "p545" "p565" "p613" "p632" "p649" "p673"
[19] "p701" "p779" "p799" "p981" "p1008" "p1131" "p1186" "p1223" "p1281"
[28] "p1383" "p1471" "p1489" "p1501" "p1565" "p1584" "p1673" "p1679" "p1782"
[37] "p1821" "p1832" "p1866" "p1891" "p1953" "p2081" "p2085" "p2140" "p2224"
[46] "p2268" "p2291" "p2384" "p2420" "p2453"

```
> summary(srdta@gtdata[vec1, 1:3])
\begin{tabular}{lrrrrrrrrr} 
& NoMeasured & CallRate & Q.2 & P.11 & P. 12 & P. 22 & Pexact & Fmax \\
rs10 & 48 & 0.96 & 0.1354167 & 36 & 11 & 1 & 1.0000000 & 0.02131603 \\
rs18 & 47 & 0.94 & 0.2765957 & 25 & 18 & 4 & 0.7245853 & 0.04298643 \\
rs29 & 45 & 0.90 & 0.1555556 & 32 & 12 & 1 & 1.0000000 & -0.01503759
\end{tabular}
            Plrt Chromosome
rs10 \(0.8843626 \quad 1\)
rs18 \(0.7697067 \quad 1\)
rs29 0.91889431

Let us explore the object returned by summary function when applied to snp.data class in more details:
```

> a <- summary(srdta@gtdata[vec1, 1:3])
> class(a)

```
[1] "data.frame"

Thus, the object returned is a data.frame. Therefore it should have dimensions and names:
```

> dim(a)

```
[1] 310
> names (a)
\begin{tabular}{lllll} 
[1] & "NoMeasured" & "CallRate" & "Q.2" & "P.11"
\end{tabular}\(\quad\) "P.12"

Indeed, we derived 8 characteristics ("NoMeasured", "CallRate", "Q.2", "P.11", "P.12", "P.22", "Pexact", "Chromosome") for the first 3 SNPs.

\section*{Exercise 11.}

Test if Hardy-Weinberg equilibrium holds for the first 10 SNPs
1. Total sample
2. In cases (bt is 1 )
3. In controls (bt is 0 )

Let us analyse the distribution of call rate in the whole study. For this, we first need to obtain the vector of call rates:
> sumgt <- summary(srdta@gtdata)
> crate <- sumgt[, "CallRate"]
This vector may be presented by a histogram
```

> hist(crate)

```
which shows that most SNPs have call rate between 93 and \(97 \%\) (figure 4.2).
As next step, you would like to produce a summary table, showing how many markers had call rate lower than, say, \(93 \%\), between 93 and \(95 \%\), between 95 and \(99 \%\) and more than \(99 \%\). You can use catable() command for that:
```

> catable(crate, c(0.93, 0.95, 0.99))

```
\begin{tabular}{lrrrr} 
& \(X<=0.93\) & \(0.93<X<=0.95\) & \(0.95<X<=0.99\) & \(X>0.99\) \\
No & 0 & 415.000 & 418.000 & 0 \\
Prop & 0 & 0.498 & 0.502 & 0
\end{tabular}

Similar procedure may be applied to see deviation from HWE:
```

> hwp <- sumgt[, "Pexact"]
> catable(hwp, c((0.05/srdta@gtdata@nsnps), 0.01, 0.05, 0.1))
X<=6.00240096038415e-05 6.00240096038415e-05<X<=0.01 0.01<X<=0.05
No 2.000 7.000 23.000
Prop 0.002 0.008 0.028
0.05<X<=0.1 X>0.1
No }31.000770.00
Prop 0.037 0.924

```

Histogram of crate


Figure 4.2: Histogram of the call rate


Figure 4.3: Histogram of the call rate

The first cut-off category will detect SNPs which are deviating from HWE at the Bonferroni-corrected P-level.

However, for these data it will make more sense to table cumulative distribution:
```

> catable(hwp, c((0.05/srdta@gtdata@nsnps), 0.01, 0.05, 0.1), cum = T)
X<=6.00240096038415e-05 X<=0.01 X<=0.05 X<=0.1 all X
No $\quad 2.000 \quad 9.000 \quad 32.00063 .000 \quad 833$
Prop 0.002
0.011
0.038
0.076
1

```

If you would like to investigate the minor allele frequency (MAF) distribution, the same logic would apply. First, derive MAF with
```

> afr <- sumgt[, "Q.2"]
> maf <- pmin(afr, (1 - afr))

```

Next, generate histograms for frequency and MAF:
```

> par(mfcol = c(2, 1))
> hist(afr)
> hist(maf)

```
(shown at the figure 4.3) and then generate table describing frequency distribution:
```

> catable(afr, c(0.01, 0.05, 0.1, 0.2, 0.5, 0.8, 0.9, 0.95, 0.99))
X<=0.01 0.01<X<=0.05 0.05<X<=0.1 0.1<X<=0.2 0.2<X<=0.5 0.5< X <=0.8
No }\begin{array}{lllllll}{22.000 53.000 }\&{99.000}\&{132.000}\&{313.000}\&{187.000}
Prop 0.026 0.064 0.119 0.158 0. 0.376 0.224
0.8<X<=0.9 0.9<X<=0.95 0.95<X<=0.99 X>0.99
No 18.000 8.00 1.000 0
Prop 0.022 0.01 0.001 0
> catable(maf, c(0, 0.01, 0.05, 0.1, 0.2), cum = T)
X<=0 X<=0.01 X<=0.05 X<=0.1 X<=0.2 all X
No 0}\quad0\quad22.000 76.000 183.00 333.0 833

```


Note that we used " 0 " as the first category - this will give you the number of monomorhic SNPs which we recommend to exclude from analysis.

Other function, perid.summary, produces summary SNP statistics per person. Let us try producing this summary for the first 10 people:
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline & NoMeasured & NoPoly & Hom & E (Hom) & Var & F & Callpp \\
\hline p1 & 790 & 707 & 0.7987342 & 0.6600319 & 0.4048662 & 0.40798616 & 0.9483794 \\
\hline p2 & 792 & 714 & 0.7474747 & 0.6585152 & 0.5090002 & 0.26050805 & 0.9507803 \\
\hline p3 & 783 & 700 & 0.6206897 & 0.6618209 & 0.4332890 & -0.12162558 & 0.9399760 \\
\hline p4 & 789 & 705 & 0.6070976 & 0.6601276 & 0.5251900 & -0.15602916 & 0.9471789 \\
\hline p5 & 790 & 707 & 0.6658228 & 0.6619821 & 0.5288936 & 0.01136232 & 0.9483794 \\
\hline p6 & 787 & 703 & 0.7662008 & 0.6622227 & 0.3770418 & 0.30783027 & 0.9447779 \\
\hline p7 & 794 & 709 & 0.6309824 & 0.6587669 & 0.4527349 & -0.08142388 & 0.9531813 \\
\hline p8 & 793 & 711 & 0.7023960 & 0.6587232 & 0.5163296 & 0.12796887 & 0.9519808 \\
\hline p9 & 788 & 711 & 0.6675127 & 0.6573272 & 0.5599395 & 0.02972375 & 0.9459784 \\
\hline p10 & 797 & 713 & 0.6587202 & 0.6614644 & 0.4889042 & -0.00810600 & 0.9567827 \\
\hline \multicolumn{8}{|c|}{Het} \\
\hline p1 & 0.2012658 & & & & & & \\
\hline p2 & 0.2525253 & & & & & & \\
\hline p3 & 0.3793103 & & & & & & \\
\hline p4 & 0.3929024 & & & & & & \\
\hline p5 & 0.3341772 & & & & & & \\
\hline p6 & 0.2337992 & & & & & & \\
\hline p7 & 0.3690176 & & & & & & \\
\hline p8 & 0.2976040 & & & & & & \\
\hline p9 & 0.3324873 & & & & & & \\
\hline p10 & 0.3412798 & & & & & & \\
\hline
\end{tabular}

This table lists the number of genotypes scored for the person, call rate, and heterozygosity. The outliers who have increased average heterozygosity may be suggestive of contaminated DNA samples.

Let us analyse the distribution of heterozygosity:
```

> het <- perid.summary(srdta)\$Het
> mean(het)

```

Histogram of het


Figure 4.4: Histogram of heterozygosity
[1] 0.3309457
```

> catable(het, c(0.1, 0.25, 0.3, 0.35, 0.5))

```
\begin{tabular}{lrrrrrr} 
& \(X<=0.1\) & \(0.1<X<=0.25\) & \(0.25<X<=0.3\) & \(0.3<X<=0.35\) & \(0.35<X<=0.5\) & \(X>0.5\) \\
No & 7.000 & 73.000 & 339.000 & 1281.000 & 800.00 & 0 \\
Prop 0.003 & 0.029 & 0.136 & 0.512 & 0.32 & 0
\end{tabular}

The resulting histogram is presented in figure 4.4 It is easy to see that few people have very low heterozygosity, but there are no outliers with extremely high values.

In this section, we covered low-level functions summary and perid.summary. Base on these, an upper-level genetic data quality control function, check.marker, is based. That function will be covered in the next section.

\section*{Summary:}
- When summary () function is applied to an object of snp.data-class, it return summary statistics for SNPs, including exact test for HardyWeinberg equilibrium.
- When perid.summary() function is applied to an object of snp.dataclass, it return per-person summary statistics, including the call rate within this person and its' heterozygosity.

\section*{Exercise 12.}

Characterise the distribution of call rates within study subjects and produce a histogram.
How many people have call rate below \(93 \%\) ?

\section*{Chapter 5}

\section*{Genome-wide association analysis}

In the first parts of this section you will be guided through a GWA analysis of a small data set. In the last part you will investigate a larger data set by yourself, do a verification study and will answer the questions. All data sets used assume a study in a relatively homogeneous population. Try to finish the first part in the morning and the second part in the afternoon.

Though only few thousands of markers located at four small chromosomes are used in the scan, we still going to call it Genome-Wide (GW), as the amount of data we will use is approaches the amount to be expected in a real experiment. However, because the regions are small, and the LD between SNPs is high, some specific features (e.g. relatively high residual inflation, which occurs because large proportion of SNPs are in LD with the reuly associated ones) are specific features of this data set, which are not observed in true GWA studies.

Start R and load GenABEL library by typing
> library (GenABEL)
and load the data which we will use in this section by
> data (ge03d2ex)
Investigate the objects loaded by command
> 1 s ()
[1] "ge03d2ex"
The ge03d2ex is an object of the class gwaa.data:
> class (ge03d2ex)
[1] "gwaa.data"
attr(,"package")
[1] "GenABEL"
To check what are the names of variables in the phenotypic data frame, use
```

> names(ge03d2ex@phdata)

```
```

[1] "id" "sex" "age" "dm2" "height" "weight" "diet" "bmi"

```

We can attach this data frame to the R search path by
```

> attach(ge03d2ex@phdata)

```

\subsection*{5.1 Data descriptives and first round of GWA analysis}

Let us investigate what are the traits presented in the data frame loaded and what are the characteristics of the distribution by using specific GenABEL function descriptive.trait:
```

> descriptives.trait(ge03d2ex)

```
\begin{tabular}{lrrr} 
& No & Mean & SD \\
id & 136 & NA & NA \\
sex & 136 & 0.529 & 0.501 \\
age & 136 & 49.069 & 12.926 \\
dm2 & 136 & 0.632 & 0.484 \\
height & 135 & 169.440 & 9.814 \\
weight & 135 & 87.397 & 25.510 \\
diet & 136 & 0.059 & 0.236 \\
bmi & 135 & 30.301 & 8.082
\end{tabular}

You can see that phenotypic frame contains the data on 136 people; the data on sex, age, height, weight, diet and body mass index (BMI) are available. Our trait of interest is dm2 (type 2 diabetes). Note that every single piece of information in this data set is simulated; however, we tried to keep our simulations in a way we think the control of T2D may work.

You can produce a summary for cases and controls separately and compare distributions of the traits by
```

> descriptives.trait(ge03d2ex, by = dm2)

```
\begin{tabular}{lrrrrrrrrrr} 
& No (by.var=0) & Mean & SD No (by.var=1) & Mean & SD & Ptt & Pkw \\
id & & 50 & NA & NA & 86 & NA & NA & NA & NA \\
sex & & 50 & 0.420 & 0.499 & 86 & 0.593 & 0.494 & 0.053 & 0.052 \\
age & & 50 & 47.038 & 13.971 & 86 & 50.250 & 12.206 & 0.179 & 0.205 \\
dm2 & 50 & NA & NA & 86 & NA & NA & NA & NA \\
height & & 49 & 167.671 & 8.586 & 86 & 170.448 & 10.362 & 0.097 & 0.141 \\
weight & & 49 & 76.534 & 17.441 & 86 & 93.587 & 27.337 & 0.000 & 0.000 \\
diet & & 50 & 0.060 & 0.240 & 86 & 0.058 & 0.235 & 0.965 & 0.965 \\
bmi & & 49 & 27.304 & 6.463 & 86 & 32.008 & 8.441 & 0.000 & 0.001 \\
& Pexact & & & & & & & & & \\
id & NA & & & & & & & & & \\
sex & 0.074 & & & & & & & & & \\
age & NA & & & & & & & & & \\
dm2 & NA & & & & & & & & &
\end{tabular}

\subsection*{5.1. DATA DESCRIPTIVES AND FIRST ROUND OF GWA ANALYSIS 7}
\begin{tabular}{lr} 
height & NA \\
weight & NA \\
diet & 1.000 \\
bmi & NA
\end{tabular}

Here, the by argument specifies the grouping variable. You can see that cases and controls are different in weight, which is expected, as T2D is associated with obesity.

Similarly, you can produce grand GW descriptives of the marker data by using
```

> descriptives.marker(ge03d2ex)
\$`Minor allele frequency distribution`
X<=0.01 0.01<X<=0.05 0.05<X<=0.1 0.1<X<=0.2 X>0.2
No 146.000 684.000 711.000 904.000 1555.000
Prop 0.036 0.171
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No }46.000\quad71.000 125.000 275.000 4000
Prop 0.012 0.018 0.031 0.069 1
\$`Distribution of porportion of successful genotypes (per person)`
X<=0.9 0.9<X<==0.95 0.95<X<=0.98 0.98<X<=0.99 X>0.99

| No 1.000 | 0 | 0 | 135.000 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Prop 0.007 0 0 0
\$`Distribution of porportion of successful genotypes (per SNP)
X<=0.9 0.9<X<=0.95 0.95<X<=0.98 0.98<X<=0.99 X>0.99

| No | 37.000 | 6.000 | 996.000 | 1177.000 | 1784.000 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Prop | 0.009 | 0.002 | 0.249 |  |  |

\$`Mean heterozygosity for a SNP [1] 0.2582298 $`Standard deviation of the mean heterozygosity for a SNP` [1] 0.1592255 $`Mean heterozygosity for a person` [1] 0.2476507 $`Standard deviation of mean heterozygosity for a person`
[1] 0.04291038
It is of note that we can see inflation of the proportion of the tests for HWE at particular threshold, as compared to the expected. This may indicate poor genotyping quality and/or genetic stratification.
We can test the GW marker characteristics in controls by

```
```

> descriptives.marker(ge03d2ex, ids = (dm2 == 0))

```
```

> descriptives.marker(ge03d2ex, ids = (dm2 == 0))

```
```

\$`Minor allele frequency distribution`
X<=0.01 0.01<X<=0.05 0.05<X<=0.1 0.1< X<=0.2 X
No 233.000 676.000 671.000 898.000 1522.00
Prop 0.058 0.169 0. 0.168 0.224 0.38
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No 0}3.000 14.000 98.000 400
Prop 0 0.001 0.003 0.024 1
\$`Distribution of porportion of successful genotypes (per person)     X<=0.9 0.9<X<=0.95 0.95<X<=0.98 0.98<X<=0.99 X>0.99 No (lllrl $`Distribution of porportion of successful genotypes (per SNP)`     X<=0.9 0.9<X<=0.95 0.95<X<=0.98 0.98< X<=0.99 X>0.99 No 37.000 49.000 1523.000 0 2391.000 Prop 0.009 0.012 0.381 0.0.5 $`Mean heterozygosity for a SNP` [1] 0.2555009 $`Standard deviation of the mean heterozygosity for a SNP` [1] 0.1618707 $`Mean heterozygosity for a person` [1] 0.2525720 $`Standard deviation of mean heterozygosity for a person`
[1] 0.04714886

```

Apparently, HWE distribution holds better in controls than in the total sample.
Let us check whether there are indications that deviation from HWE is due to cases. At this stage we are only interested in HWE distribution table, and therefore will ask to report only table two:
```

> descriptives.marker(ge03d2ex, ids = (dm2 == 1))[2]
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No 45.000 79.00 136.000 268.000 4000
Prop 0.011 0.02 0.034 0.067 1

```

It seems that indeed excessive number of markers are out of HWE in cases. If no laboratory procedure (e.g. DNA extraction, genotyping, calling) were done for cases and controls separately, this may indicate possible heterogeneity specific for cases.

It may be interesting to plot a \(\chi^{2}-\chi^{2}\) plot contrasting observed and expected distributions for the test for HWE in cases. First, we need to compute summary SNP statistics by
```

> s <- summary(ge03d2ex@gtdata[(dm2 == 1), ])

```

Note the you have produced the summary for the gtdata slot of ge03d2ex; this is the slot which actually contain all genetic data in special compressed format.

You can see first 10 elements of this very long table by
```

> s[1:10,]

```
\begin{tabular}{lrrrrrrrr} 
& NoMeasured & CallRate & Q.2 P.11 & P.12 & P. 22 & Pexact \\
rs7435137 & 84 & 0.9767442 & 0.52380952 & 17 & 46 & 21 & 0.510978370 \\
rs7725697 & 85 & 0.9883721 & 0.01176471 & 83 & 2 & 0 & 1.000000000 \\
rs664063 & 86 & 1.0000000 & 0.08720930 & 71 & 15 & 0 & 1.000000000 \\
rs4670072 & 60 & 0.6976744 & 0.11666667 & 53 & 0 & 7 & 0.001701645 \\
rs546570 & 84 & 0.9767442 & 0.89880952 & 1 & 15 & 68 & 1.000000000 \\
rs7908680 & 83 & 0.9651163 & 0.03012048 & 78 & 5 & 0 & 1.000000000 \\
rs166732 & 83 & 0.9651163 & 0.04216867 & 76 & 7 & 0 & 1.000000000 \\
rs4257079 & 86 & 1.0000000 & 0.07558140 & 73 & 13 & 0 & 1.000000000 \\
rs5150804 & 84 & 0.9767442 & 0.39880952 & 31 & 39 & 14 & 0.820496827 \\
rs3508821 & 83 & 0.9651163 & 0.20481928 & 52 & 28 & 3 & 1.0000000000
\end{tabular}
    Fmax Plrt Chromosome
rs7435137-0.09772727 0.3699602726 1
rs7725697-0.01190476 0.8773691192 3
rs664063 -0.09554140 0.2308999066 2
rs4670072 1.000000000 .0002510899 X
rs546570 0.01830931 0.8693645189 2
rs7908680 -0.03105590 0.6935168932 1
rs166732 -0.04402516 0.5787401988 1
rs4257079 -0.08176101 0.3022863648 1
rs5150804 \(0.03177183 \quad 0.7710793180 \quad 2\)
rs3508821 -0.03565062 0.74195874062

Note that the column before the last provides P-exact we need. We can extract these to a separate vector by

100100
Let us first try do GWA scan using the raw (before quality control) data. We will use the score test, as implemented in the qtscore() funcrtion of GenABEL for testing:
```

> an0 <- qtscore(dm2, ge03d2ex, trait = "binomial")

```

The first argument used describes the model; here it is rather simple - the affection status, dm2, is supposed to depend on SNP genotype only.

You can see what objects are returned by this function by using
```

> names(an0)

```
\begin{tabular}{rllll} 
[1] "chi2.1df" & "chi2.2df" & "P1df" & "P2df" & "Pc1df" \\
[6] "Pc2df" & "lambda" & "effB" & "effAB" & "effBB" \\
[11] "N" & "snpnames" & "idnames" & "map" & "chromosome" \\
[16] "formula" & "family" & & &
\end{tabular}

Here, P1df, P2df and Pc1df are most interesting; the first two are vectors of 1 and 2 d.f. P-values obtained in the GWA analysis, the last one is 1 d.f. P-value corrected for inflation factor \(\lambda\) (which is presented in lambda object).

Let us see if there is evidence for the inflation of the test statistics
```

> anO\$lambda

```
\$estimate
[1] 1.029658
\$se
[1] 0.0005343007
\$iz0
[1] 1
\$iz2
[1] 1

The estimate of \(\lambda\) is 1.03 , suggesting inflation of the test and some degree of stratification.

The \(\lambda\) is computed by regression in a Q-Q plot. Both estimation of \(\lambda\) and production of the \(\chi^{2}-\chi^{2}\) plot can be done using the estlambda function:
```

> estlambda(an0\$P1df)
\$estimate
[1] 1.029658
\$se
[1] 0.0005343007

```

The corresponding \(\chi^{2}-\chi^{2}\) plot is presented in Figure 5.1.

The 'se' produced by estlambda can not be used to test if inflation is significant and make conclusions about presence of stratification.

We can also present the obtained results using the "Manhatten plot", where a SNP genomic position is on the X -axes and \(-\log _{10}\) of the \(p\)-value is shown on Y-axes:
```

> plot(an0)

```

The resulting plot is presented in the figure 5.2. By default, \(-\log _{10}(P-v a l u e)\) of not corrected 1 d.f. test are presented; see help to figure out how this behaviour can be changed.

We can also add the corrected P-values to the plot with
```

> add.plot(an0, df = "Pc1df", col = c("lightblue", "lightgreen"))

```

You can also generate a descriptive table for the "top" (as ranked by P-value) results by


Figure 5.1: \(\chi^{2}-\chi^{2}\) plot for a GWA scan. Black line of slope 1: expected under no inflation; Red line: fitted slope.
```

> descriptives.scan(an0)

```
\begin{tabular}{lrrrrrr} 
& Chromosome & Position & N & effB & P1df & Pc1df \\
rs1719133 & 1 & 4495479 & 136 & 0.33729339 & 0.0002795623 & 0.0003425105 \\
rs2975760 & 3 & 10518480 & 134 & 3.80380024 & 0.0002983731 & 0.0003649107 \\
rs7418878 & 1 & 2808520 & 136 & 3.08123060 & 0.0009743183 & 0.0011540593 \\
rs5308595 & 3 & 10543128 & 133 & 3.98254950 & 0.0010544366 & 0.0012463049 \\
rs4804634 & 1 & 2807417 & 132 & 0.43411456 & 0.0011970132 & 0.0014100096 \\
rs3224311 & 2 & 6009769 & 135 & 3.15831710 & 0.0013290907 & 0.0015611949 \\
rs26325 & 3 & 10617781 & 135 & 0.09742793 & 0.0013313876 & 0.0015638203 \\
rs8835506 & 2 & 6010852 & 132 & 3.17720829 & 0.0015321522 & 0.0017928676 \\
rs3925525 & 2 & 6008501 & 135 & 2.98416931 & 0.0019400358 & 0.0022558582 \\
rs2521089 & 3 & 10487652 & 135 & 2.50239493 & 0.0020524092 & 0.0023829357 \\
& effAB & effBB & P2df & & \\
rs1719133 & 0.4004237 & 0.0000000 & 0.0006333052 & & \\
rs2975760 & 3.4545455 & 10.0000000 & 0.0011434877 & & \\
rs7418878 & 3.6051282 & 4.8717949 & 0.0022642036 & & \\
rs5308595 & 3.3171429 & & Inf & 0.0045930101 & & \\
rs4804634 & 0.5240642 & 0.1739130 & 0.0036964462 & & \\
rs3224311 & 3.4151786 & 4.2500000 & 0.0029405999 & & \\
rs26325 & 0.1097724 & NA & 0.0013313876 & & \\
rs8835506 & 3.4903846 & 4.1250000 & 0.0031618340 & & &
\end{tabular}

\section*{qtscore(dm2, ge03d2ex, binomial)}


Figure 5.2: \(-\log _{10}(P-\) value \()\) from the genome scan before QC procedure. Raw analysis: black; corrected analysis: green
```

rs3925525 3.2380952 4.1212121 0.0045554384
rs2521089 2.5717703 4.7727273 0.0069661425

```

Here you see top 10 results, sorted by P-value with 1 d.f. If you want to sort by the corrected P-value, you can use descriptives.scan (an0, sort="Pc1df"); to see more then 10 (e.g. 25) top results, use descriptives. \(\operatorname{scan}(a n 0\), top=25). You can combine all these options. Large part of results reports NA as effect estimates and 9.99 as \(P\)-value for 2 d.f. test - for these markers only two out of three possible genotypes were observed, and consequently 2 d.f. test could not be performed.

Now let us apply qtscore() function with times argument, which tells it to compute empirical GW (or experiment-wise) significance
```

> an0.e <- qtscore(dm2, ge03d2ex, times = 200, quiet = TRUE)

```
```

> descriptives.scan(an0.e, sort = "Pc1df")
Chromosome Position N effB P1df Pc1df effAB effBB
rs1719133 1 4495479 136 -0.2652064 0.385 0.490 -0.2080882 -0.7375000
rs2975760 3 10518480 134 0.2340655 0.415 0.510 0.2755102 0.4090909
rs7418878 1 2808520}136 0.2089098 0.885 0.910 0.2807405 0.3268398

```
```

rs5308595 3 10543128 133 0.2445516 0.895 0.935 0.2564832 0.4623656
rs4804634 1 2807417 132 -0.2050449 0.925 0.955 -0.1193830-0.3845238
rs8835506 2 6010852 132 0.2112000 0.960 0.960 0.2796221 0.3076923
rs3224311 2 6009769 135 0.2133633 0.945 0.960 0.2778634 0.3151515
rs26325
rs3925525 2 6008501 135 0.2057095 0.985 0.995 0.2660834 0.3074627
3 10617781 135 -0.4875367 0.950 0.960 -0.4875367 NA
rs2521089 3 10487652 135
P2df
rs1719133 9.99
rs2975760 9.99
rs7418878 9.99
rs5308595 9.99
rs4804634 9.99
rs8835506 9.99
rs3224311 9.99
rs26325 9.99
rs3925525 9.99
rs2521089 9.99

```

None of the SNPs hits GW significance. If, actually, any did pass the threshold, we could not trust the results, because the distribution of the HWE test and presence of inflation factor for the association test statistics suggest that the data may contain multiple errors (indeed they do). Therefore before association analysis we need to do rigorous Quality Control (QC).

Note that at certain SNP, corrected \(P\)-values become equal to 1 - at this point order is arbitrary because sorting could not be done.

\section*{Summary:}
- The descriptives family of functions was developed to facilitate production of tables which can be directly used in a manuscript - it is possible to save the output as a file, which can be open by Excel or Word. See e.g. help(descriptives.trait) for details.
- The inflation of test statistics compared to null (1 d.f.) may be estimated with estlambda function.

\subsection*{5.2 Genetic data QC}

The major genetic data QC function of GenABEL is check.marker(). We will try to run it; the output is rather self-explaining. As it was detailed at the lecture, in the first round of the QC we do not want to check for HWE. This can be achieved by setting HWE P-value selection threshold to zero (p.level=0):
```

> qc1 <- check.marker(ge03d2ex, p.level = 0)
Excluding people/markers with extremely low call rate...
4000 markers and 136 people in total

```
```

O people excluded because of call rate < 0.1
6 markers excluded because of call rate < 0.1
Passed: 3994 markers and 136 people
Running sex chromosome checks...
197 heterozygous X-linked male genotypes found
1 X-linked markers are likely to be autosomal (odds > 1000 )
2 male are likely to be female (odds > 1000 )
O female are likely to be male (odds > 1000 )
If these people/markers are removed, O heterozygous male genotypes are left
Passed: }3993\mathrm{ markers and }134\mathrm{ people
no X/Y/mtDNA-errors to fix

```

RUN 1
3993 markers and 134 people in total
304 ( \(7.613323 \%\) ) markers excluded as having low ( \(<1.865672 \%\) ) minor allele frequency
36 ( \(0.9015778 \%\) ) markers excluded because of low ( \(<95 \%\) ) call rate
0 ( \(0 \%\) ) markers excluded because they are out of HWE ( \(\mathrm{P}<0\) )
1 ( \(0.7462687 \%\) ) people excluded because of low ( \(<95 \%\) ) call rate
Mean autosomal HET is 0.2747262 (s.e. 0.03721277 )
3 ( \(2.238806 \%\) ) people excluded because too high autosomal heterozygosity (FDR <1\%)
Excluded people had HET >= 0.4856887
Mean IBS is 0.7711323 (s.e. 0.02132557), as based on 2000 autosomal markers
1 ( \(0.7462687 \%\) ) people excluded because of too high IBS (>=0.95)
In total, 3653 ( \(91.4851 \%\) ) markers passed all criteria
In total, 129 ( \(96.26866 \%\) ) people passed all criteria

RUN 2
3653 markers and 129 people in total
72 ( \(1.970983 \%\) ) markers excluded as having low ( \(<1.937984 \%\) ) minor allele frequency
\(0(0 \%)\) markers excluded because of low ( \(<95 \%\) ) call rate
0 ( \(0 \%\) ) markers excluded because they are out of HWE ( \(\mathrm{P}<0\) )
0 ( \(0 \%\) ) people excluded because of low ( \(\langle 95 \%\) ) call rate
Mean autosomal HET is 0.2744972 (s.e. 0.01706096)
0 people excluded because too high autosomal heterozygosity (FDR <1\%)
Mean IBS is 0.7695853 (s.e. 0.0179941 ), as based on 2000 autosomal markers
0 ( \(0 \%\) ) people excluded because of too high IBS ( \(>=0.95\) )
In total, 3581 ( \(98.02902 \%\) ) markers passed all criteria
In total, 129 ( \(100 \%\) ) people passed all criteria
RUN 3
3581 markers and 129 people in total
0 ( \(0 \%\) ) markers excluded as having low ( \(<1.937984 \%\) ) minor allele frequency
0 ( \(0 \%\) ) markers excluded because of low ( \(<95 \%\) ) call rate
0 ( \(0 \%\) ) markers excluded because they are out of HWE ( \(\mathrm{P}<0\) )
0 ( \(0 \%\) ) people excluded because of low ( \(\langle 95 \%\) ) call rate
Mean autosomal HET is 0.2744972 (s.e. 0.01706096)
0 people excluded because too high autosomal heterozygosity (FDR <1\%)

Mean IBS is 0.7703515 (s.e. 0.01804976 ), as based on 2000 autosomal markers
0 ( \(0 \%\) ) people excluded because of too high IBS ( \(>=0.95\) )
In total, 3581 ( \(100 \%\) ) markers passed all criteria
In total, 129 ( \(100 \%\) ) people passed all criteria

The computation of all pairwise proportion of alleles identical-by-state (IBS) by ibs() function, which is also called by check.markers() may take quite some time, which is proportional to the square of the number of subjects. This is not a problem with the small number of people we use for this example or when modern computers are used. However, the computers in the Nihes computer room are very old. Therefore be prepared to wait for long time when you will do a self-exercise with 1,000 people.

From the output you can see that QC starts with checking the data for SNPs and people with extremely low call rate. Six markers are excluded from further analysis due to very low call rate. Next, X-chromosomal errors are identified. The function finds out that all errors (heterozygous male X-genotypes) are due to two people with wrong sex assigned and one marker, which looks like an autosomal one. This actually could be a marker from pseudoautosomal region, which should have been arranged as a separate "autosome".

Then, the procedure finds the markers with low call rate \((\leq 0.95)\) across people, markers with low MAF (by default, low MAF is defined as less than few copies of the rare allele, see help for details); people with low call rate ( \(\leq 0.95\) ) across SNPs, people with extreme heterozygosity (at FDR 0.01) and these who have GW IBS \(\geq 0.95\). These default parameters may be changed if you wish (consult help).

Because some of the people fail to pass the tests, the data set is not guaranteed to be really "clean" after single iteration, e.g. some marker may not pass the call threshold after we exclude few informative (but apparently wrong) people. Therefore the QC is repeated iteratively until no further errors are found.

You can generate short summary of QC by marker and by person through
```

> summary(qc1)

```
\$` Per-SNP fails statistics`
        NoCall NoMAF NoHWE Redundant Xsnpfail
\begin{tabular}{lrrrrr} 
NoCall & 42 & 0 & 0 & 0 & 0 \\
NoMAF & NA & 376 & 0 & 0 & 0 \\
NoHWE & NA & NA & 0 & 0 & 0 \\
Redundant & NA & NA & NA & 0 & 0 \\
Xsnpfail & NA & NA & NA & NA & 1
\end{tabular}
\$’Per-person fails statistics`
        IDnoCall HetFail IBSFail isfemale ismale isXXY
\begin{tabular}{lrrllll} 
IDnoCall & 1 & 0 & 0 & 0 & 0 & 0 \\
HetFail & NA & 3 & 0 & 0 & 0 & 0 \\
IBSFail & NA & NA & 1 & 0 & 0 & 0
\end{tabular}
\begin{tabular}{lllllll} 
isfemale NA NA NA & NA & 0 & 0 & 0
\end{tabular}
\begin{tabular}{lrrrrrr} 
ismale & NA & NA & NA & NA & 0 & 0 \\
isXXY & NA & NA & NA & NA & NA & 0
\end{tabular}

Note that the original data, ge03d2ex, are not modified during the procedure; rather, check.markers () generate a list of markers and people which pass or do not pass certain QC criteria. The objects returned by check.markers() are:
```

> names(qc1)

```
```

[1] "nofreq" "nocall" "nohwe" "Xmrkfail" "hetfail" "idnocall"
[7] "ibsfail" "isfemale" "ismale" "snpok" "idok" "call"

```

The element idok provides the list of people who passed all QC criteria, and snpok provides the list of SNPs which passed all criteria. You can easily generate a new data set, which will consist only of these people and markers by > data1 <- ge03d2ex[qc1\$idok, qc1\$snpok]

If there are any residual sporadic X-errors (male heterozygosity), these can be fixed (set to NA) by
```

> data1 <- Xfix(data1)
no X/Y/mtDNA-errors to fix

```

Applying this function does not make any difference for the example data set, but you will need to use it for the bigger data set.

At this point, we are ready to work with the new, cleaned, data set data1. However, if we try
> length (dm2)
[1] 136
we can see that the original phenotypic data are attached to the search path (there are only 129 people left in the 'clean' data set). Therefore we need to detach the data by
```

> detach(ge03d2ex@phdata)

```
and attach new data by
```

> attach(data1@phdata)

```

At this stage, let us check if the first round of QC solves the problem of inflated test for HWE, which may be the case if this inflation is due to genotypic errors we managed to eliminate:
```

> descriptives.marker(data1)[2]
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No }43.000\quad65.000 121.000 240.000 358
Prop 0.012 0.018 0.034 0.067 1
> descriptives.marker(data1[dm2 == 1])[2]
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No 46.000 70.00 127.000 228.000 3581
Prop 0.013 0.02 0.035 0.064 1

```

\subsection*{5.3 Finding genetic sub-structure}

Now, we are ready for the second round of QC, detection of genetic outliers which may contaminate our data. We will detect genetic outliers using a technique, which resembles the one suggested by Price at al.

As a first step, we will compute a matrix of genomic kinship between all pairs of people, using only autosomal markers by
```

> data1.gkin <- ibs(data1[, data1@gtdata@chromosome != "X"], weight = "freq")

```

You can see the \(5 \times 5\) upper left sub-matrix by
```

> data1.gkin[1:5, 1:5]

```
\begin{tabular}{lrrrrr} 
& id199 & id300 & id403 & id415 & id666 \\
id199 & 0.489655824 & 3262.00000000 & 3261.00000000 & 3249.00000000 & 3265.0000000 \\
id300 & -0.012267995 & 0.48794278 & 3268.00000000 & 3257.00000000 & 3271.0000000 \\
id403 & -0.012464681 & -0.01262302 & 0.51160501 & 3255.00000000 & 3270.0000000 \\
id415 & -0.002399026 & 0.01363777 & -0.02528089 & 0.56091781 & 3259.0000000 \\
id666 & -0.019116287 & -0.02110468 & 0.02096914 & -0.02025885 & 0.5620686
\end{tabular}

This step may take few minutes on large data sets or when using old computers!

The numbers below the diagonal show genomic kinship (IBD), the numbers on the diagonal correspond to 0.5 plus the genomic homozigosity, and the numbers above the diagonal tell how many SNPs were typed successfully for both subjects (thus the IBD estimate is derived using this number of SNPs).

Second, we transform this matrix to a distance matrix using standard R command
```

> data1.dist <- as.dist(0.5 - data1.gkin)

```

Finally, we perform Classical Multidimensional Scaling by
```

> data1.mds <- cmdscale(data1.dist)

```
by default, the first two principal components are computed and returned.

This may take few minutes on large data sets or when using old computers!

We can present the results graphically by
```

> plot(data1.mds)

```

The resulting plot is presented in figure 5.3. Each point on the plot corresponds to a person, and the 2D distances between points were fitted to be as close as possible to these presented in the original IBS matrix. You can see that study subjects clearly cluster in two groups.

You can identify the points belonging to clusters by
```

> km <- kmeans(data1.mds, centers = 2, nstart = 1000)
> cl1 <- names(which(km$cluster == 1))
> cl2 <- names(which(km$cluster == 2))
> cl1

```
[1] "id2097" "id6954" "id2136" "id858"
> cl2
```

    [1] "id199" "id300" "id403" "id415" "id666" "id689" "id765" "id830"
    [9] "id908" "id980" "id994" "id1193" "id1423" "id1505" "id1737" "id1827"
    [17] "id1841" "id2068" "id2094" "id2151" "id2317" "id2618" "id2842" "id2894"
[25] "id2985" "id3354" "id3368" "id3641" "id3831" "id3983" "id4097" "id4328"
[33] "id4380" "id4395" "id4512" "id4552" "id4710" "id4717" "id4883" "id4904"
[41] "id4934" "id4961" "id5014" "id5078" "id5274" "id5275" "id5454" "id5853"
[49] "id5926" "id5969" "id6237" "id6278" "id6352" "id6501" "id6554" "id6663"
[57] "id6723" "id7499" "id7514" "id7541" "id7598" "id7623" "id7949" "id8059"
[65] "id8128" "id8281" "id8370" "id8400" "id8433" "id8772" "id8880" "id8890"
[73] "id8957" "id8996" "id9082" "id9901" "id9930" "id1857" "id2528" "id4862"
[81] "id9184" "id5677" "id6407" "id5472" "id2135" "id8545" "id4333" "id1670"
[89] "id1536" "id6917" "id6424" "id3917" "id9628" "id9635" "id4729" "id5190"
[97] "id6399" "id6062" "id620" "id1116" "id6486" "id41" "id677" "id4947"
[105] "id9749" "id6428" "id7488" "id5949" "id2924" "id5783" "id4096" "id903"
[113] "id9049" "id185" "id1002" "id362" "id9014" "id5044" "id2749" "id5437"
[121] "id2286" "id4743" "id4185" "id8330" "id6934"

```

Four outliers are presented in the smaller cluster.

Now you will need to use the BIGGER cluster for to select study subjects. Whether this will be cl1 or cl2 in you case, is totally random.

We can form a data set which is free from outliers by using only people from the bigger cluster:
```

> data2 <- data1[cl2, ]

```

After we dropped the outliers, we need to repeat QC using check.markers(). At this stage, we want to allow for HWE checks (we will use only controls and exclude markers with \(\mathrm{FDR} \leq 0.2\) ):
```

> qc2 <- check.marker(data2, hweids = (data2@phdata\$dm2 == 0),

+ fdr = 0.2)
Excluding people/markers with extremely low call rate...
3 5 8 1 markers and 1 2 5 people in total
0 people excluded because of call rate < 0.1
O markers excluded because of call rate < 0.1
Passed: }3581\mathrm{ markers and }125\mathrm{ people
Running sex chromosome checks...

```


Figure 5.3: Mapping samples on the space of the first two Principle Components resulting from analysis of genomic kinship. Red dots identify genetic outliers
```

O heterozygous X-linked male genotypes found
O X-linked markers are likely to be autosomal (odds > 1000 )
O male are likely to be female (odds > 1000 )
O female are likely to be male (odds > 1000 )
If these people/markers are removed, 0 heterozygous male genotypes are left
Passed: 3581 markers and 125 people
no X/Y/mtDNA-errors to fix
RUN 1
3581 markers and 125 people in total
40 (1.117006%) markers excluded as having low (<2%) minor allele frequency
0 (0%) markers excluded because of low (<95%) call rate
0 (0%) markers excluded because they are out of HWE (FDR <0.2)
0 (0%) people excluded because of low (<95%) call rate
Mean autosomal HET is 0.2776868 (s.e. 0.01655360)
0 people excluded because too high autosomal heterozygosity (FDR <1%)
Mean IBS is 0.7702688 (s.e. 0.01256862), as based on 2000 autosomal markers
0 (0%) people excluded because of too high IBS (>=0.95)
In total, 3541 (98.883%) markers passed all criteria
In total, 125 (100%) people passed all criteria

```
```

RUN 2
3541 markers and 125 people in total
0 (0%) markers excluded as having low (<2%) minor allele frequency
0 (0%) markers excluded because of low (<95%) call rate
0 (0%) markers excluded because they are out of HWE (FDR <0.2)
0 (0%) people excluded because of low (<95%) call rate
Mean autosomal HET is 0.2776868 (s.e. 0.01655360)
O people excluded because too high autosomal heterozygosity (FDR <1%)
Mean IBS is 0.7727586 (s.e. 0.01245084), as based on 2000 autosomal markers
0 (0%) people excluded because of too high IBS (>=0.95)
In total, 3541 (100%) markers passed all criteria
In total, 125 (100%) people passed all criteria
> summary(qc2)
\$`Per-SNP fails statistics`
NoCall NoMAF NoHWE Redundant Xsnpfail

| NoCall | 0 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| NoMAF | NA | 40 | 0 | 0 | 0 |
| NoHWE | NA | NA | 0 | 0 | 0 |
| Redundant | NA | NA | NA | 0 | 0 |
| Xsnpfail | NA | NA | NA | NA | 0 |

\$`Per-person fails statistics`
IDnoCall HetFail IBSFail isfemale ismale isXXY
IDnoCall 0 0 0 0 0
HetFail NA 0 0
IBSFail NA NA 0 N 0
isfemale NA NA NA 0
ismale NA NA NA NA 0
isXXY NA NA NA NA NA 0

```

If the procedure did not run, check previous Note.

Indeed, in the updated data set few markers do not pass our QC criteria and we need to drop a few markers. This is done by
```

> data2 <- data2[qc2$idok, qc2$snpok]

```

This is going to be our final analysis data set, therefore let us attach the phenotypic data to the search path, then we do not need to type data2@phdata\$... to access dm2 status or other variables:
```

> detach(data1@phdata)
> attach(data2@phdata)

```

\subsection*{5.4 GWA association analysis}

Let us start again with descriptives of the phenotypic and marker data
```

> descriptives.trait(data2, by = dm2)

|  | No (by.var=0) | Mean | SD No(by.var=1) | Mean | SD | Ptt | Pkw |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| id |  | 48 | NA | NA | 77 | NA | NA | NA | NA |
| sex |  | 48 | 0.438 | 0.501 | 77 | 0.597 | 0.494 | 0.084 | 0.082 |
| age |  | 48 | 46.378 | 13.865 | 77 | 50.593 | 12.465 | 0.089 | 0.097 |
| dm2 |  | 48 | NA | NA | 77 | NA | NA | NA | NA |
| height |  | 47 | 167.988 | 8.610 | 77 | 170.423 | 10.646 | 0.166 | 0.223 |
| weight |  | 47 | 77.273 | 17.427 | 77 | 94.160 | 26.963 | 0.000 | 0.000 |
| diet |  | 48 | 0.062 | 0.245 | 77 | 0.065 | 0.248 | 0.957 | 0.957 |
| bmi |  | 47 | 27.485 | 6.539 | 77 | 32.235 | 8.335 | 0.001 | 0.001 |
|  | Pexact |  |  |  |  |  |  |  |  |
| id | NA |  |  |  |  |  |  |  |  |
| sex | 0.098 |  |  |  |  |  |  |  |  |
| age | NA |  |  |  |  |  |  |  |  |
| dm2 | NA |  |  |  |  |  |  |  |  |
| height | NA |  |  |  |  |  |  |  |  |

```

You can see that relation to weight is maintained in this smaller, but hopefully cleaner, data set; moreover, relation to age becomes boundary significant.

If you check descriptives of markers (only HWE part shown)
> descriptives.marker(data2)[2]
\$` Cumulative distr. of number of SNPs out of HWE, at different alpha` \(\mathrm{X}<=1 \mathrm{e}-04 \mathrm{X}<=0.001 \mathrm{X}<=0.01 \mathrm{X}<=0.05\) all X
\begin{tabular}{llllll} 
No & 1 & 2.000 & 22.000 & 108.00 & 3541
\end{tabular}
\begin{tabular}{llllll} 
Prop & 0 & 0.001 & 0.006 & 0.03 & 1
\end{tabular}
you can see that the problems with HWE are apparently fixed; we may guess that these were caused by the Wahlund's effect.

Run the score test on the cleaned data by
```

> data2.qt <- qtscore(dm2, data2)

```
and check lambda
```

> data2.qt\$lambda
\$estimate
[1] 1.040969
\$se
[1] 0.0007325815

```
\$iz0
qtscore(dm2, data2)


Figure 5.4: \(-\log _{10}(\) Corrected \(P\) - value \()\) from the genome scan after the QC procedure
[1] 1.016119

\section*{\$iz2}
[1] 1
there is still some inflation, which is explained by the fact that we investigate only few short chromosomes with high LD and few causative variants.

Produce the association analysis plot by
```

> plot(data2.qt, df = "Pc1df")
(figure 5.4).
Produce the scan summary by
> descriptives.scan(data2.qt, sort = "Pc1df")

|  | Chromosome | Position | N | effB | P1df | Pc1df |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| rs1719133 | 1 | 4495479 | 125 | -0.2709715 | 0.0003599745 | 0.000470699 |
| rs8835506 | 2 | 6010852 | 122 | 0.2346446 | 0.0008465818 | 0.001072548 |
| rs4804634 | 1 | 2807417 | 122 | -0.2161383 | 0.0010949169 | 0.001374104 |
| rs3925525 | 2 | 6008501 | 125 | 0.2280847 | 0.0011081904 | 0.001390146 |
| rs3224311 | 2 | 6009769 | 125 | 0.2280847 | 0.0011081904 | 0.001390146 |
| rs2975760 | 3 | 10518480 | 124 | 0.2244898 | 0.0012545339 | 0.001566573 |
| rs4534929 | 1 | 4474374 | 124 | -0.1864322 | 0.0019996882 | 0.002454805 |

```
```

rs6079246 2 7048058 124 -0.4730491 0.0021060217 0.002580453
rs5308595 3 10543128 123 0.2731788 0.0023674963 0.002888472
rs1013473 1 4 4487262 125 0.1864494 0.0025659783 0.003121460
effAB effBB P2df
rs1719133-0.2153891 -0.7260274 0.0009290908
rs8835506 0.3201507 0.3201507 0.0012972329
rs4804634-0.0971660 -0.3947368 0.0026493765
rs3925525 0.3046200 0.3187614 0.0020102814
rs3224311 0.3046200 0.3187614 0.0020102814
rs2975760 0.2570423 0.3959311 0.0047945922
rs4534929 -0.1487634-0.3968254 0.0074296050
rs6079246 -0.4730491 NA 0.0021060217
rs5308595 0.2844101 0.4719101 0.0095505795
rs1013473 0.2714833 0.3782654 0.0067821374

```

Comparison with the top 10 from the scan before QC shows that results changed substantially with only few markers overlapping.

You can see similar results when accessing empirical GW significance:
```

> data2.qte <- qtscore(dm2, data2, times = 200, quiet = TRUE)

```
```

> descriptives.scan(data2.qte, sort = "Pc1df")

```
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline & Chromosome & Position & N & effB & P1df & Pc1df & eff AB & \(f \mathrm{fB}\) \\
\hline rs1719133 & 1 & 4495479 & 125 & -0.2709715 & 0.480 & 0.600 & -0.2153891 & -0.7260274 \\
\hline rs8835506 & 2 & 6010852 & 122 & 0.2346446 & 0.800 & 0.870 & 0.3201507 & 0.3201507 \\
\hline rs4804634 & 1 & 2807417 & 122 & -0.2161383 & 0.870 & 0.925 & -0.0971660 & -0.3947368 \\
\hline rs3925525 & 2 & 6008501 & 125 & 0.2280847 & 0.870 & 0.925 & 0.3046200 & 0.3187614 \\
\hline rs3224311 & 2 & 6009769 & 125 & 0.2280847 & 0.870 & 0.925 & 0.3046200 & 0.3187614 \\
\hline rs2975760 & 3 & 10518480 & 124 & 0.2244898 & 0.900 & 0.960 & 0.2570423 & 0.3959311 \\
\hline rs5308595 & 3 & 10543128 & 123 & 0.2731788 & 0.995 & 0.995 & 0.2844101 & 0.4719101 \\
\hline rs6079246 & 2 & 7048058 & 124 & -0.4730491 & 0.970 & 0.995 & -0.4730491 & NA \\
\hline rs1013473 & 1 & 4487262 & 125 & 0.1864494 & 0.995 & 0.995 & 0.2714833 & 0.3782654 \\
\hline rs8258863 & 1 & 4735725 & 124 & -0.3787795 & 0.995 & 0.995 & -0.3787795 & NA \\
\hline
\end{tabular}
    P2df
rs1719133 0.680
rs8835506 0.805
rs4804634 0.970
rs3925525 0.935
rs3224311 0.935
rs2975760 0.995
rs5308595 1.000
rs6079246 1.000
rs1013473 1.000
rs8258863 1.000

Again, none of the SNPs hits GW 5\% significance. Still, you can see that after QC top markers achieve somewhat "better" significance.

In the last part, we will do several adjusted and stratified analyses. Only empirical P-values will be estimated to make the story shorter. To adjust for sex and age, we can
```

> data2.qtae <- qtscore(dm2 ~ sex + age, data2, times = 200, quiet = T)
> descriptives.scan(data2.qtae)
Chromosome Position N effB P1df Pc1df effAB effBB
rs1719133 1 4495479 125 -0.2589016 0.605 0.700 -0.22773582 -0.6210215
rs8835506 2 6010852 122 0.2221511 0.930 0.970 0.31550163 0.2814815
rs3925525 2 6008501 125 0.2187193 0.950 0.980}00.30498571 0.2825598
ls3224311 2 6009769 125 0.2187193 0.950 0.980
rs4804634 1 2 2807417 122 -0.2065161 0.960 0.980 -0.09071397 -0.3764928
rs6079246 2 7048058 124 -0.4687637 0.980 0.995 -0.46876373 NA
rs2939190 1 71843 125 -0.2151462 0.995 0.995 -0.27260946 -0.3173458
rs1037237 3 11690145 125 0.1701179 0.995 1.000 0.09349904 0.3326606
rs4534929 1 4474374 124 -0.1709477 0.995 1.000-0.14282148 -0.3597864
rs2398949 1 1 4828375 123-0.3185459 0.995 0.995 -0.41947254 0.2730503
r
rs1719133 0.920
rs8835506 0.810
rs3925525 0.910
rs3224311 0.910
rs4804634 0.990
rs6079246 1.000
rs2939190 1.000
rs1037237 1.000
rs4534929 1.000
rs2398949 0.775

```

You can see that there is little difference between adjusted and unadjusted analysis, but this is not always the case; adjustment may make your study much more powerful when covariates explain a large proportion of environmental trait variation.

Finally, let us do stratified (by BMI) analysis. We will contracts obese ( \(B M I \geq 30\) ) cases to all controls.
```

> data2.qtse <- qtscore(dm2 ~ sex + age, data2, ids = ((bmi > 30 \&

+ dm2 == 1) | dm2 == 0), times = 200, quiet = TRUE)

```
\begin{tabular}{lrrrrrrr} 
> descriptives.scan(data2.qtse, sort \(=" P c 1 d f ") ~\) \\
& Chromosome & Position & \(N\) & effB & P1df & Pc1df & effAB \\
rs1891586 & 1 & 2297398 & 89 & -0.25255255 & 0.795 & 0.810 & -0.18449217 \\
rs794264 & 1 & 2534738 & 88 & 0.22850926 & 0.920 & 0.940 & 0.31845203 \\
rs9178808 & 1 & 2536431 & 89 & -0.22656534 & 0.955 & 0.960 & -0.16522870 \\
rs5032886 & 1 & 2537020 & 89 & -0.22656534 & 0.955 & 0.960 & -0.16522870 \\
rs9630764 & 1 & 3897972 & 89 & 0.22317446 & 0.970 & 0.970 & 0.21102609 \\
rs7504607 & 1 & 2704056 & 89 & -0.45829946 & 0.970 & 0.970 & -0.45829946 \\
rs2884479 & \(X\) & 13618173 & 87 & -0.20852009 & 0.985 & 0.985 & -0.44885734 \\
rs7435137 & 1 & 4259040 & 89 & 0.08104670 & 1.000 & 1.000 & 0.12984966
\end{tabular}
```

rs664063 2 7288020 89 -0.06490513 1.000 1.000 -0.01598230
rs546570 2 6120257 89 -0.12391227 1.000 1.000 0.10998954
effBB P2df
rs1891586 -0.47441121 1.0
rs794264 0.49475804 1.0
rs9178808 -0.49835345 1.0
rs5032886 -0.49835345 1.0
rs9630764 0.44206126 1.0
rs7504607 NA 1.0
rs2884479 -0.32984151 0.4
rs7435137 0.15517100 1.0
rs664063 -0.51438083 1.0
rs546570 -0.01392273 1.0

```

Again, noting interesting at GW significance level. If we would have had found something, naturally, we would not known if we mapped a T2D or obesity gene (or a gene for obesity in presence of T2D, or the one for T2D in presence of obesity).

At this point, you acquired the knowledge necessary for the self-exercise. Please close R by \(q\) () command and proceed to the next section.

\subsection*{5.5 GWA exercise}

During the exercise, you will work with a larger data set (approximately 1,000 people and \(7,000+\) SNPs). You are to do complete three-round QC; perform GWA analysis with dm2 as the outcome of interest and identify 10 SNPs which you would like to take to the stage 2 (replication) scan. You will do replication analysis using a confirmatory data set. If you did everything right, the SNPs which you identified as significant or replicated will be located in know T2D genes.

Please keep in mind that the data are simulated, and do not take your findings too seriously!

Start R by going to "Start \(->\) Programs \(->\) R \(->\) R-2.4.1". Load GenABEL library by
```

> library(GenABEL)

```

The two data sets we will use in this exercise are part of the GenABEL distribution. The first one ("discovery" set) can be loaded by
```

> data(ge03d2)

```

Please move along the lines detailed in the guided exercise and try to answer following questions:

Exercise 15 How many cases and controls are presented in the original data set?

Exercise 16 How many markers are presented in the original data set?
Exercise 17 Is there evidence for inflation of the HWE test staistics?

Exercise 18 Analyse empirical \(G W\) significance. How many SNPs pass genomewide significance threshold, after correction for the inflation factor? Write down the names of these SNPs for further comparison.

Perform complete three steps of the genetic data QC.
Exercise 19 How many male turned apparently female?
Exercise 20 How many sporadic \(X\) errors do you still observe even when the female male and non-X X-markers are removed? (do not forget to Xfix() these!)

Exercise 21 How many "twin" DNAs did you discover?
Exercise 22 How many genetic outliers did you discover?
After you have finished QC, answer the questions:
Exercise 23 How many cases and controls are presented in the data after \(Q C\) ?
Exercise 24 How many markers are presented in the data after \(Q C\) ?
Exercise 25 Is there evidence for inflation of the HWE test staistics?
Exercise 26 Perform GWA analysis of the cleaned data, using assimptotic test and plot the results. What is the estimate of \(\lambda\) for the 1 d.f. test?

Exercise 27 Analyse empirical \(G W\) significance. How many SNPs pass genomewide significance threshold, after correction for the inflation factor? Do these SNPs overlap much with the ones ranked at the top before the QC? If not, what could be the reason?

If time permits, do analysis with adjustment for covariates and stratified analysis.

Select 10 SNPs which you would like to follow-up. Say, you've selected rs1646456, rs7950586, rs4785242, rs4435802, rs2847446, rs946364, rs299251, rs2456488, rs1292700, and rs8183220.

Make a vector of these SNPs with
```

> vec12 <- c("rs1646456", "rs7950586", "rs4785242", "rs4435802",

+ "rs2847446", "rs946364", "rs299251", "rs2456488", "rs1292700",
+ "rs8183220")

```

Load the stage 2 (replicaton) data set by
```

> data(ge03d2c)

```
and select the subset of SNPs you need by
```

> confdat <- ge03d2c[, vec12]

```

Analyse the confdat for association with dm2.
Exercise 28 Given the two-stage design, and applying the puristic criteria specified in the lecture, for how many SNPs you can claim a significant finding?

Exercise 29 Using the same criteria, for how many SNPs you can claim a replicated finding?

You can check if any of the SNPs you have identified as significant or replicated are the ones which were simulated to be associated with dm2 by using the command
```

> show.ncbi(c("snpname1", "snpname2", "snpname3"))

```
where snpnameX stands for the name of your identified SNP. The "true" SNPs can be found on NCBI and are located in known T2D genes (just because we used these names to name the "significant" ones).

If time permits, characterise the mode of inheritance of the significant SNPs. You can convert data from GenABEL format to the format used by dgc. genetics and genetics libraries by using as.genotype() function. Consult help for details. Please do not attempt to convert more then few dozens SNPs: the format of genetics is not compressed, which means conversion may take long and your low-memory computer may even crash if you attempt to convert the whole data set.

If time permits, try to do first round of QC allowing for HWE checks (assume FDR of 0.1 for total sample). In this case, can you still detect stratification in the "cleaned" data?

\section*{Chapter 6}

\section*{GWA analysis in presence of stratification: theory}

In genetic association studies, we look for association between a genetic polymorphism and the value of a trait of interest. The best scenario - the one we always hope for - is that the observed association results from causation, that is the polymorphism studied is functionally involved in the control of the trait. However, association has no direction, and making causal inference in epidemiology in general and in genetic epidemiology in particular is usually not possible based on statistical analysis only.

In fact, most associations observed in genetic studies are due to a confounder - an (unobserved) factor which is associated with both the genetic polymorphism and the trait analysed. Presence of such factor leads to induced, "secondary" correlation between the trait and the polymorphism; if we would have controlled for that factor in the association model, the relation between the polymorphism and the trait would have gone.

100
There are two major types of confounders leading to induced correlation in genetic association studies. One type is "good" confounding of association by the real, unobserved functional variant, which is, as a rule, not present on the SNP array, but is in linkage disequilibrium (LD) with typed SNP. Under this scenario, the functional variant is associated with the trait because of causative relation; at the same time it is associated with a typed polymorphism located nearby because of LD. This confounding induces secondary correlation between the typed polymorphism and the trait, making localistion of the true functional polymorphism (LD mapping) possible.

Other major type of confounding observed in genetic association studies is confounding by population (sub)structure. Let us consider a study in which subjects come from two distantly related populations, say Chinese and European. Due to genetic drift, these two populations will have very different frequencies at many loci throughout the genome. At the same time, these two populations are different phenotypically (prevalence of different disease, mean value of quantitative traita) due to accumulated genetic and cultural differences. Therefore any of these traits will show association with multiple genomic loci. While some of these associations may be genuine genetic associations in a sense that ei-
ther the polymorphisms themselves, or the polymorphisms close by are causally involved, most of these associations will be genetically false positives - noise associations generated by strong genetic and phenotypic divergence between the two populations.

The scenario described above is extreme and indeed it is hard to imagine a genetic asociation study in which two very distinct populations are so blintly mixed and analysed not taking this mixture into account. However, a more subtle scenario where several slighly genetically different populations are mixed in the same study is frequently the case and a matter of concern in GWA studies.

In this chapter, we will define what is genetic structure, and how it can be quantified (section 6.1); what are the effects of genetic structure on the standard association tests (section 6.2) and specific association tests which take possible genetic structure into account (section 6.3).

\subsection*{6.1 Genetic structure of populations}

A major unit of genetic structure is a genetic population. Different definitions of genetic population are available, for example Wikipedia defines population (biol.) as "the collection of inter-breeding organisms of a particular species". The genetics of populations is "the study of the allele frequency distribution and change under the influence of ...evolutionary processes". In the framework of population genetics, the main characteristics of interest of a group of individuals are their genotypes, frequencies of alleles in this group, and the dynamics of these distributions in time. While the units of interest of population genetics are alleles, the units of evolutionary processes are acting upon are organisms. Therefore a definition of a genetic population should be based on the chance that different alleles, present in the individuals in question can mix together; if such chance is zero, we may consider such groups as different populations, each described by its own genotypic and allelic frequencies and their dynamic. Based on these considerations, a genetic population may be defined a in the following way:

Two individuals, \(I_{1}\) and \(I_{2}\), belong to the same population if (a) the probability that they would have an offspring in common is greater then zero and (b) this probability is much higher than the probability of \(I_{1}\) and \(I_{2}\) having an offspring in common with some individual \(I_{3}\), which is said to belong to other genetic population.

Here, to have an offspring in common does not imply having a direct offspring, but rather a common descendant in a number of generations.

However, in gene discovery in general and GWA studies in particular we are usually not interested in future dinamics of alleles and genotypes distributions. What is the matter of concern in genetic association studies is potential common ancestry - that is that individuals may share common ancestors and thus share in common the alleles, which are exact copies of the same ancestral allele. Such alleles are called "identical-by-descent", or IBD for short. If the chance of IBD is high, this reflects high degree of genetic relationship. As a rule, relatives share many features, both environmental and genetic, which may lead to confounding.

Genetic relationship between a pair of individuals is quantified using the "coefficient of kinship", which measures that chance that gametes, sampled at random from these individuals, are IBD.

Thus for the purposes of gene-discovery we can define genetic population use retrospective terms and based on the concept of IBD:

Two individuals, \(I_{1}\) and \(I_{2}\), belong to the same genetic population if (a) their genetic relationship, measured with the coefficient of kinship, is greater then zero and (b) their kinship is much higher than kinship between them and some individual \(I_{3}\), which is said to belong to other genetic population.

One can see that this definition is quantitative and rather flexible (if not to say arbitrary): what we call a "population" depends on the choice of the threshold for the "much-higher" probability. Actually, what you define as "the same" genetic population depends in large part on the scope aims of your study. In human genetics literature you may find references to a particular genetically isolated population, population of some country (e.g. "German population", "population of United Kingdom"), European, Caucasoid or even general human population. Defining a population is about deciding on some probability threshold.

In genetic association studies, it is frequently assumed that study participants are "unrelated" and "come from the same genetic population". Here, "unrelated" means, that while study participants come from the same population (so, there is non-zero kinship between them!), this kinship is so low that it has very little effect on the statistical testing procedures used to study association between genes and phenotypes.

In the following sections we will consider the effects of population structure on the istribution of genotypes in a study population. We will start with assumption of zero kinship between study participants, which would allow us to formulate Hary-Weinberg principle (section 6.1.1). In effect, there is no such thing as zero kinship between any two organisms, however, when kinship is very low, the effects of kinship on genotypic distribution are minimal, as we will see in section 6.1.2. The effects of substructure - that is when study sample consist of several genetic populations - onto genotypic distribution will be considered in section 6.1.3. Finally, we will generalize the obtained results for the case of arbitrary structures and will see what are the effects of kinship onto joint distribution of genotypes and phenotypes in section ??.

\subsection*{6.1.1 Hardy-Weinberg equilibrium}

To describe genetic structure of populations we will use rather simplistic model approximating genetic processes in natural populations. Firstly, we will assume that the population under consideration has infinitely large size, which implies that we can work in terms of probabilities, and no random process take place. Secondly, we accept non-overlapping
\[
\text { generation } \Rightarrow \text { gametic pool } \Rightarrow \text { generation }
\]
model. This model assumes that a set of individuals contributes gametes to genetic pool, and dies out. The gametes are sampled randomly from this pool in pairs to form individuals of the second generation. The selection acts on individuals, while mutation occurs when the gametic pool is formed. The key point of this model is the abstract of gametic pool: if you use that, you do not need to consider all pair-wise mating between male and female individuals; you rather
consider some abstract infinitely large pool, where gametes are contributed to with the frequency proportional to that in previous generation. Interestingly, this rather artificial construct has a great potential to describe the phenomena we indeed observe in nature.

In this section, we will derive Hardy-Weinberg low (this analog of the Mendel's low for populations). The question to be answered is, if some alleles at some locus segregate according to Mendel's lows and aggregate totally at random, what would be genotypic distribution in a population?

Let us consider two alleles, wild type normal allele \((N)\) and a mutant \((D)\), segregating at some locus in the population and apply the "generation \(\Rightarrow\) gametic pool \(\Rightarrow\) generation" model. Let us denote the ferquency of the \(D\) allele in the gametic pool as \(q\), and the frequency of the other allele, \(N\), as \(p=1-q\). Gametes containing alleles \(N\) and \(D\) are sampled at random to form diploid individuals of the next generation. The probability to sample a " \(N\) " gamete is \(p\), and the probability that the second sampled gamete is also " \(N\) " is also \(p\). According to the rule, which states that joint probability of two independent events is a product of their probabilities, the probability to sample " \(N\) " and " \(N\) " is \(p \cdot p=p^{2}\). In the same manner, the probability to sample " \(D\) " and then " \(D\) " is \(q \cdot q=q^{2}\). The probability to sample first the mutant and then normal allele is \(q \cdot p\), the same is the probability to sample " \(D\) " first and " \(N\) " second. In most situations, we do not (and can not) distinguish heterozygous genotypes \(D N\) and \(N D\) and refer to both of them as " \(N D\) ". In this notation, frequency of \(N D\) will be \(q \cdot p+p \cdot q=2 \cdot p \cdot q\). Thus, we have computed the genotypic distribution for a population formed from a gametic pool in which the frequency of \(D\) allele was \(q\).

To obtain the next generation, the next gametic pool is generated. The frequency of \(D\) in the nect gametic pool is \(q^{2}+\frac{1}{2} \cdot 2 \cdot p \cdot q\). Here, \(q^{2}\) is the probability that a gamete-contributing individual has genotype \(D D ; 2 \cdot p \cdot q\) is the probability that a gamete-contributing individual is \(N D\), and \(\frac{1}{2}\) is the probability that \(N D\) individual contributes \(D\) allele (only half of the gametes contributed by individuals with \(N D\) genotype are \(D\) ); see Figure 6.1. Thus the freqeuncy of \(D\) in the gametic pool is \(q^{2}+\frac{1}{2} \cdot 2 \cdot p \cdot q=q \cdot(q+p)=q\) - exactly the same as it was in previous gametic pool.

Thus, if assumptions of random segregation and aggregation hold, the expected frequency of \(N N, N D\) and \(D D\) genotypes are stable over generations and can be related to the allelic frequencies using the follwoing relation
\[
\begin{array}{ll}
P(N N)=(1-q) \cdot(1-q) & =p^{2}, \\
P(N D)=q \cdot(1-q)+(1-q) \cdot q & =2 \cdot p \cdot q,  \tag{6.1}\\
P(D D)=q \cdot q & \\
=q^{2}
\end{array}
\]
which is known as Hardy-Weinberg equlibrium (HWE) point.
There are many reasons, in which random segregation and aggregation, and, consequently, Hardy-Weinberg equilibrium, are violated. It is very important to realize that, especially if the study participants are believed to come from the same genetic population, most of the times when deviation from HWE is detected, this deviation is due to technical reasons, i.e. genotyping error. Therefore testing for HWE is a part of the genotypic quality control procedure in most studies. Only when the possibility of technical errors is eliminated, other possible explanations may be considered. In a case when deviation from HWE can


\section*{Alleles}

Figure 6.1: Genotypic and allelic frequency distribution in a population; \(q=\) \(P(D)=P(D D)+\frac{1}{2} \cdot P(D N)\).
not be explained by technical reasons, the most frequent explanation would be that the sample tested is composed of representatives of different genetic populations, or more subtle genetic structure. However, unless study participants represent a mixture of very distinct genetic populations - the chances of which coming unnoticed are low - the efffects of genetic structure on HWE are difficult to detect, at least for any single marker, as you will see in the next sections.

\subsection*{6.1.2 Inbreeding}

Inbreeding is preferential breeding between (close) relatives. An extreme example of inbreeding is a selfing, a breeding system, observed in some plants. The inbreeding is not uncommon in animal and human populations. Here, the main reason for inbreeding are usually geographical (e.g. mice live in very small interbred colonies - dems - which are usually established by few mice and are quite separated from other dems) or cultural (e.g. noble families of Europe).

Clearly, such preferential breeding between relatives violates the assumption of random aggregation, underling Hardy-Weinberg principle. Relatives are likely to share the same alleles, inherited from common ancestors. Therefore their progeny has an increased chance of being autozygous - that is to inherit a copy of exactly the same ancestral allele from both parents. An autozygous genotype is always homozygous, therefore inbreeding should increase the frequency of homozygous, and decrease the frequency of heterozygous, genotypes.

Inbreeding is quantified by the coefficient of inbreeding, which is defined as the probability of autozygosity. This coefficient may characterize an individual, or a population in general, in which case this is expectation that a random individual from the population is autozygous at a random locus. The coefficient of inbreeding is closely related to the coefficient of kinship, defined earlier for a pair of individuals as the probability that two alleles sampled at random from these individuals, are IBD. It is easy to see that the coefficient of inbreeding for a person is the same as the kinship between its parents.


Figure 6.2: Inbred family structure (A) and probability of individual " \(G\) " being autozygous for the "Red" ancestral allele

Let us compute the inbreeding coefficient for the person \(\mathbf{J}\) depicted at figure \(6.2 \mathbf{J}\) is a child of \(\mathbf{G}\) and \(\mathbf{H}\), who are cousins. \(\mathbf{J}\) could be autozygous at for example "red" allele of founder grand-grand-parent A, which could have been transmitted through the meioses \(\mathbf{A} \Rightarrow \mathbf{D}, \mathbf{D} \Rightarrow \mathbf{G}\), and \(\mathbf{G} \Rightarrow \mathbf{J}\), and also through the path \(\mathbf{A} \Rightarrow \mathbf{E}, \mathbf{E} \Rightarrow \mathbf{H}\), and \(\mathbf{H} \Rightarrow \mathbf{J}\) (Figure \(6.2 \mathbf{B}\) ). What is the chance for \(\mathbf{J}\) to be autozygous for the "red" allele? The probability that this particular founder allele is transmitted to \(\mathbf{D}\) is \(1 / 2\), the same is the probability that the allele is transmitted from \(\mathbf{D}\) to \(\mathbf{G}\), and the probability that the allele is transmitted from \(\mathbf{G}\) to \(\mathbf{J}\). Thus the probability that the "red" allele is transmitted from \(\mathbf{A}\) to \(\mathbf{J}\) is \(1 / 2 \cdot 1 / 2 \cdot 1 / 2=1 / 2^{3}=1 / 8\). The same is the chance that that allele is transmitted from \(\mathbf{A}\) to \(\mathbf{E}\) to \(\mathbf{H}\) to \(\mathbf{J}\), therefore the probability that \(\mathbf{J}\) would be autozygous for the red allele is \(1 / 2^{3} \cdot 1 / 2^{3}=1 / 2^{6}=1 / 64\). However, we are interested in autozygosity for any founder allele; and there are four such alleles ("red", "green", "yellow" and "blue", figure 6.2 B). For any of these the probability of autozygosity is the same, thus the total probability of autozygosity for \(\mathbf{J}\) is \(4 \cdot 1 / 64=1 / 2^{4}=1 / 16\).

Now we shall estimate the expected genotypic probability distribution for a person characterized with some arbitrary coefficient of inbreeding, \(F\) - or for a population in which average inbreeding is \(F\). Consider a locus with two alleles, \(A\) and \(B\), with frequency of \(B\) denoted as \(q\), and frequency of \(A\) as \(p=1-q\). If the person is autozygous for some founder allele, the founder allele may be either \(A\), leading to autozygous genotype \(A A\), or the founder allele may be \(B\), leading to genotype \(B B\). The chance that the founder allele is \(A\) is \(p\), and the chance that the founder allele is \(B\) is \(q\). If the person is not autozygous, then the expected genotypic frequencies follow HWE. Thus, the probability of genotype \(A A\) is \((1-F) \cdot p^{2}+F \cdot p\), where the first term corresponds to probability that the person is \(A A\) given it is not inbred \(\left(p^{2}\right)\), multiplied by the probability that it is not inbred \((1-F)\), and the second term corresponds to probability that a


Figure 6.3: Genotypic probability distribution for a locus with \(50 \%\) frequency of the \(B\) allele; black bar, no inbreeding; red, \(F=0.001\); green, \(F=0.01\); blue, \(F=0.05\)
person is \(A A\) given it is inbred \((p)\), multiplied by the probability that the person is inbred \((F)\). This computations can be easily done for all genotypic classes leading to the expression for HWE under inbreeding.
\[
\begin{array}{ll}
P(A A)=(1-F) \cdot p^{2}+F \cdot p & =p^{2}+p \cdot q \cdot F \\
P(A B)=(1-F) \cdot 2 \cdot p \cdot q+F \cdot 0 & =2 \cdot p \cdot q \cdot(1-F)  \tag{6.2}\\
P(B B)=(1-f) \cdot q^{2}+F \cdot q & =q^{2}+p \cdot q \cdot F
\end{array}
\]

How much is inbreeding expected to modify genotypic distribution in human populations? The levels of inbreeding observed in human genetically isolated populations typically vary between 0.001 (low inbreeding) to 0.05 (relatively high), see (1) (2). The genotypic distribution for \(q=0.5\) and different values of the inbreeding coefficient is shown at the figure 6.3

What is the power to detect deviation from HWE due to inbreeding? For that, we need to estimate the expectation of the \(\chi^{2}\) statistics (the non-centrality parameter, NCP) used to test for HWE. The test for HWE is performed using
standard formula
\[
\begin{equation*}
T^{2}=\sum_{i} \frac{\left(O_{i}-E_{i}\right)^{2}}{E_{i}} \tag{6.3}
\end{equation*}
\]
where summation is performed over all classes (genotypes); \(O_{i}\) is the count observed in \(i\)-th class, and \(E_{i}\) is the count expected under the null hypothesis (HWE). Under the null hypothesis, this test statistic is distributed as \(\chi^{2}\) with number of degrees of freedom equal to the number of genotypes minus the number of alleles.

Thus the expectation of this test statistic for some \(q, F\), and \(N\) (sample size) is
\[
\begin{align*}
E\left[T^{2}\right] & =\frac{\left(N\left(q^{2}+p q F\right)-N q^{2}\right)^{2}}{N q^{2}}+\frac{(N 2 p q(1-F)-N 2 p q)^{2}}{N 2 p q}+\frac{\left(N\left(p^{2}+p q F\right)-N p^{2}\right)^{2}}{N p^{2}} \\
& =\frac{(N p q F)^{2}}{N q^{2}}+\frac{(-2 N p q F)^{2}}{N 2 q q}+\frac{(N p q F)^{2}}{N p^{2}} \\
& =N p^{2} F^{2}+2 N p q F^{2}+N q^{2} F^{2}  \tag{6.4}\\
& =N F^{2}\left(p^{2}+2 p q+q^{2}\right) \\
& =N \cdot F^{2}
\end{align*}
\]

Interestingly, the non-centrality parameter does not depend on the allelic frequency. Given the non-centrality parameter, it is easy to compute the power to detect deviation from HWE for any given \(F\). For example, to achieve the power of \(>0.8\) at \(\alpha=0.05\), for a test with one degree of freedom the noncentrality parameter should be \(>7.85\). Thus, if \(F=0.05\), to have \(80 \%\) power, \(N \cdot F^{2}>7.85\), that is the required sample size should be \(N>\frac{7.85}{F^{2}}=\frac{7.85}{0.0025}=\) 3140 people.

Thus, even in populations with strong inbreeding, rather large sample sizes are required to detect the effects of inbreeding on HWE at a particular locus, even at relatively weak significance level of \(5 \%\).

While the chance that deviation from HWE due to inbreeding will be statistically significant is relatively small, inbreeding may have clear effects on the results of HWE testing in GWA study. Basically, if testing is performed at a threshold corresponding to nominal significance \(\alpha\), a proportion of markers which show significant deviation will be larger than \(\alpha\). Clearly, how large this proportion will be depends on the inbreeding and on size of the study - expectation of \(T^{2}\) is a function of both \(N\) an \(F\). A proportion of markers showing significant deviation form HWE at different values of inbreeding, sample size, and nominal significance threshold, is shown in table 6.1. While deviation of this proportion from nominal one is minimal at large \(\alpha\) 's and small sample sizes and coefficients of inbreeding, it may be 10 -fold and even 100 -fold higher than the nominal level at reasonable values of \(N\) and \(F\) for smaller thresholds.

\subsection*{6.1.3 Mixture of genetic populations: Wahlund's effect}

Consider the following artificial example. Imagine that recruitment of study participants occurs at a hospital, which serves two equally size villagec ( \(V_{1}\) and \(\left(V_{2}\right)\); however, the villages are very distinct because of cultural reasons, and most marriages occur within a village. Thus these two villages represent two genetically distinct populations. Let us consider a locus with two alleles, \(A\) and

Table 6.1: Expected proportion of markers deviating from HWE in a sample of \(N\) people coming from a population with average inbreeding \(F\). Proportion of markers is shown for particular test statistic threshold, corresponding to nominal significance \(\alpha\).
\begin{tabular}{lcccc}
\hline & & \multicolumn{3}{c}{\(\alpha\)} \\
\cline { 3 - 5 }\(N\) & \(F\) & 0.05 & \(10^{-4}\) & \(5 \cdot 10^{-8}\) \\
\hline \multirow{2}{*}{1,000} & 0.001 & 0.0501 & \(1.008 \cdot 10^{-4}\) & \(5.077 \cdot 10^{-8}\) \\
& 0.005 & 0.0529 & \(1.205 \cdot 10^{-4}\) & \(7.025 \cdot 10^{-8}\) \\
& 0.010 & 0.0615 & \(1.885 \cdot 10^{-4}\) & \(14.503 \cdot 10^{-8}\) \\
\hline \multirow{3}{*}{10,000} & 0.001 & 0.0511 & \(1.081 \cdot 10^{-4}\) & \(5.784 \cdot 10^{-8}\) \\
& 0.005 & 0.0790 & \(3.544 \cdot 10^{-4}\) & \(36.991 \cdot 10^{-8}\) \\
& 0.010 & 0.1701 & \(19.231 \cdot 10^{-4}\) & \(426.745 \cdot 10^{-8}\) \\
\hline
\end{tabular}

Table 6.2: Genotypic proportions in a mixed population
\begin{tabular}{|c|c|c|c|c|c|}
\hline Village & \%Sample & \(p(A)\) & \(P(A A)\) & \(P(A B)\) & \(P(B B)\) \\
\hline \(V_{1}\) & 50 & 0.9 & 0.81 & 0.18 & 0.01 \\
\hline \(V_{2}\) & 50 & 0.2 & 0.04 & 0.32 & 0.64 \\
\hline & & & & Observed & \\
\hline \multirow[t]{5}{*}{Pooled} & \multirow[t]{5}{*}{100} & \multirow[t]{5}{*}{0.55} & 0.425 & 0.25 & 0.325 \\
\hline & & & & Expected & \\
\hline & & & 0.30 & 0.50 & 0.20 \\
\hline & & & & Difference & \\
\hline & & & 0.125 & -0.250 & 0.125 \\
\hline
\end{tabular}
\(B\). The frequency of \(A\) is 0.9 in \(V_{1}\) and it is 0.2 in \(V_{2}\). In each population, marriages occur at random, and HWE holds for the locus. What genotypic distribution is expected in a sample ascertained in the hospital, which represents a 1:1 mixture of the two populations?

The expected gentypic proportions are presented in table 6.2. First, assuming that HWE holds for each of the populations, we can compute genotypic proportions within these (rows 1 and 2 of table 6.2). If our sample represents a 1:1 mixture of these populations, then the frequency of some genotype is also 1:1 mixture of the respective frequencies. For example, frequency of \(A A\) genotype would be \(\frac{0.81}{2}+\frac{0.04}{2}=0.425\), and so on. The frequency of the \(A\) allele in pooled sample will be \(0.425+\frac{0.25}{2}=0.55\). Based on this frequency we would expect genotypic frequency distribution of \(0.3,0.5\) and 0.2 , for \(A A, A B\), and \(B B\), respectively. As you can see the observed distribution has much higher frequencies of homozygous genotypes - excess of homozygotes.

It is notable, that the differences between the observed homozygotes frequencies and these expected under HWE are both 0.125 , and, consequently, the observed heterozygosity is less than that expected by \(0.125 \cdot 2=0.25\).

The phenomenon of deviation from HWE due to the fact that considered population consist of two sub-populations, is known as "Wahlund's effect", after the scientist who has first considered and quantified genotypic distribution under such model(3).

Such marked differences between observed and expected under HWE are very easily detected; for the above example, a sample of \(\approx 35\) people is enough

Table 6.3: Genotypic proportions of \(P P A R \gamma\) Pro12Ala genotype in a mixed population
\begin{tabular}{lccccc}
\hline Ethnics & \%Sample & \(p(\) Pro \()\) & P(Pro/Pro \()\) & P(Pro/Ala \()\) & \(P(\) Ala/Ala \()\) \\
\hline Caucasian & 50 & 0.85 & 0.7225 & 0.2550 & 0.0225 \\
Afro-American & 50 & 0.99 & 0.9801 & 0.0198 & 0.0 .001 \\
& & & & Observed & \\
\hline Pooled & 100 & 0.92 & 0.8513 & \begin{tabular}{c}
0.1374 \\
\\
\cline { 4 - 5 }
\end{tabular} & \\
& & 0.8464 & 0.0113 \\
\cline { 3 - 5 } & & & 0.0049 & 0.1472 & 0.0064 \\
& & & & Difference & \\
\hline
\end{tabular}
to reject the hypothesis of HWE (power \(>80 \%\) at \(\alpha=0.05\) ).
However, the differences we can see in real life are not so marked. For example, the common Pro allele at position 12 of the peroxisome proliferator-activated receptor gamma is associated with increased risk for type 2 diabetes. The frequency of the Pro allele is about \(85 \%\) in European populations and CaucasianAmericans, about \(97 \%\) in Japan and \(99 \%\) in African-American (see table 1 from (4)). Table 6.3 shows hypothetical observed and expected genotypic proportions in a sample composed of \(50 \%\) Caucasians and \(50 \%\) African-American.

You can see that observed distribution and the one expected under HWE are very similar; only a sample as large as 1,800 people would allow detection of the deviation from HWE (power \(>80 \%\) at \(\alpha=0.05\) ). The situation is similar for most genes observed in real life - while the frequencies may be (or may be not) very different for populations, which diverged long time ago, for relatively close populations expected frequency differences are small and large sample sizes are required to detect deviation from HWE due to Wahlund's effect at a particular fixed locus.

Let us summarize, what genotypic proportions are expected in a sample, which is a mixture of two populations. Let each population is in HWE, and the frequency of the \(B\) allele is \(q_{1}\) in population one and \(q_{2}\) in population two. Let the proportion of individuals coming from population one is \(m\) in the mixed population, and consequently the proportion of individuals from population two is \((1-m)\). The allelic frequencies, and genotypic distributions in the original and mixed populations are presented in tale 6.4

The frequency of the \(B\) allele in the mixed population is just the weighted average of the allelic frequencies in the two populations, \(\bar{q}=m \cdot q_{1}+(1-m) \cdot q_{2}\). Let us denote the frequency of the \(A\) allele as \(\bar{p}=1-\bar{q}\). It can be demonstrated that the genotypic frequency distribution in the mixed sample is the function of the frequency of allele \(B\) in the sample, \(\bar{q}\), and "disequilibrium" parameter \(D\) :
\[
\begin{array}{ll}
P(A A) & =\bar{p}^{2}+\bar{p} \cdot \bar{q} \cdot F_{s t} \\
P(A B) & =2 \cdot \bar{p} \cdot \bar{q} \cdot\left(1-F_{s t}\right)  \tag{6.5}\\
P(B B) & =\bar{q}^{2}+\bar{p} \cdot \bar{q} \cdot F_{s t}
\end{array}
\]
where
\[
\begin{equation*}
F_{s t}=\frac{m \cdot(1-m) \cdot\left(q_{1}-q_{2}\right)^{2}}{\bar{p} \cdot \bar{q}} \tag{6.6}
\end{equation*}
\]

You can see that equation 6.5, expressing the genotypic frequencies distri-

Table 6.4: Expected genotypic proportions in a mixed population; \(F_{s t}\) is defined by equation 6.6
\begin{tabular}{lccccc}
\hline Population & Prop. & \(p(B)\) & \(P(A A)\) & \(P(A B)\) & \(P(B B)\) \\
\hline\(P_{1}\) & \(m\) & \(q_{1}\) & \(p_{1}^{2}\) & \(2 p_{1} q_{1}\) & \(q_{1}^{2}\) \\
\(P_{2}\) & \((1-m)\) & \(q_{2}\) & \(p_{2}^{2}\) & \(2 p_{2} q_{2}\) & \(q_{2}^{2}\) \\
& & & & Observed & \\
\hline Pooled & 1.0 & \(\bar{q}=m q_{1}\) & \(m p_{1}^{2}\) \\
& & \(+(1-m) q_{2} ;\) & \(+(1-m) p_{2}^{2} ;\) & \begin{tabular}{c}
\(2 m p_{1} q_{1}\) \\
\(+2(1-m) p_{2} q_{2} ;\) \\
\\
\end{tabular} & \\
& & & \(\bar{p}^{2}\) & \(m q_{1}^{2}\) \\
& & & Expected \\
& & & \(\overline{p q} F_{s t}\) & \(2 \overline{p q}\) & Difference \\
& & & & \(-2 \overline{p q} F_{s t}\) & \(\bar{p} q_{2}^{2} F_{s t}\) \\
\hline
\end{tabular}
bution under Wahlund's effect, is remarkably similar (actually, is specifically re-written in a form similar) to the equation 6.2, expressing the genotypic proportions under the effects of inbreeding. Again, the reason is that \(F_{s t}\) (as well as \(F\) of equation 6.2 is easily estimated from the data as the ratio between the observed and expected variances of the genotypic distributions. Then the expected non-centrality parameter for the test of HWE is simply \(N \cdot F_{s t}^{2}\), where \(N\) is the sample size. Therefore our results concerning the proportion of tests expected to pass a particular significance threshold when genome-wide data are analyzed (table 6.1) hold, with replacement of \(F\) with \(F_{s t}\).

We can compute that the values of \(F_{s t}\), corresponding to the population mixtures presented in tables 6.2 and 6.3 are 0.49 and 0.067 , respectively, which gives us a shortcut to estimate the sample size required to detect deviation from HWE due to Wahlund's effect (at \(\alpha=0.05\) and power \(80 \%\) ): \(N>7.85 / 0.49^{2} \approx\) 32 and \(N>7.85 / 0.067^{2} \approx 1771\).

A typical value of \(F_{s t}\) for European populations is about 0.002 (up to \(0.023(5)\) ); very large sample sizes are required to detect deviation from HWE at any given locus at such small \(F_{s t}\) 's. However, the effects onto the proportion of markers failing to pass HWE test in GWA may be visibly inflated (table 6.1.

\subsection*{6.2 Effects of population structure on standard tests for association}

\subsection*{6.2.1 Standard tests for genetic association}

Standard tests for association between genes and a binary trait are the test for allele frequency difference between cases and controls, and the Armitage's trend test for proportions (that the proportion of cases changes across genotypic groups). For quantitative traits, one of the standard tests is the score test for association, which is closely related - even equivalent - to the Armitage's trend test.

We will start with presenting the study data as \(2 \times 3\) table, where the rows correspond to the case/control status and columns correspond to genotypic groups, and the cells contain counts of events (table 6.5). For example, \(r_{0}\) is the number of cases with genotype \(A A, s_{0}\) is the number of controls with genotype \(A A\) and

Table 6.5: Counts of cases and controls with different genotypes
\begin{tabular}{lcccc}
\hline & \multicolumn{3}{c}{ Genotype } & \\
\cline { 2 - 4 } Status & \(A A\) & \(A B\) & \(B B\) & Total \\
\hline Case & \(r_{0}\) & \(r_{1}\) & \(r_{2}\) & \(R\) \\
Control & \(s_{0}\) & \(s_{1}\) & \(s_{2}\) & \(S\) \\
Total & \(n_{0}\) & \(n_{1}\) & \(n_{2}\) & \(N\) \\
\hline
\end{tabular}

Table 6.6: Counts of alleles in cases and controls
\begin{tabular}{lccc} 
Status & \(A\) & \(B\) & Total \\
\hline Case & \(2 \cdot r_{0}+r_{1}\) & \(r_{1}+2 \cdot r_{2}\) & \(2 \cdot R\) \\
Control & \(2 \cdot s_{0}+s_{1}\) & \(s_{1}+2 \cdot s_{2}\) & \(2 \cdot S\) \\
Total & \(2 \cdot n_{0}+n_{1}\) & \(n_{1}+2 \cdot n_{2}\) & \(2 \cdot N\) \\
\hline
\end{tabular}
so on.
This table can be re-arranged in a 2 x 2 allelic table, presented in table 6.6 . Each cell of this table contains the counts of alleles present in cases and controls, e.g. total number of \(A\) alleles in cases is \(2 \cdot r_{0}\) (twice the number of cases who are homozygous for the \(A\) allele) plus the number of \(A\) alleles present in heterozygous cases ( \(r_{1}\) ).

Based on these tables, we can test if the allelic frequency is different between the cases and controls, using standard \(\chi^{2}\) test, formulated as
\[
\begin{equation*}
T^{2}=\sum_{i} \frac{\left(O_{i}-E_{i}\right)^{2}}{E_{i}} \tag{6.7}
\end{equation*}
\]
where summation is performed over all cells (defined by combination of genotype/allele and phenotype); \(O_{i}\) is the count observed in \(i\)-th class, and \(E_{i}\) is the count expected under the null hypothesis (equal frequencies in cases and controls). Under the null hypothesis, this test statistic is distributed as \(\chi^{2}\) with number of degrees of freedom equal to the number of independent classes.

The null hypothesis assumes that the frequency of the \(A\) allele is the same in both cases and controls, and is equal to the frequency for \(A\) in the total sample: \(\bar{p}=\frac{2 \cdot n_{0}+n_{1}}{2 \cdot N}\). Thus the expected count of \(A\) in cases is \(2 \cdot R \cdot \bar{p}\), and the expected count of \(B\) alleles is \(2 \cdot R \cdot(1-\bar{p})\). Similarly, for cases, the expected count of \(A\) is \(2 \cdot S \cdot \bar{p}\), and the expected count of \(B\) is \(2 \cdot S \cdot(1-\bar{p})\).

Now, for the table 6.6 we can re-write the allelic test as
\[
\begin{aligned}
T_{A}^{2}= & \frac{\left(\left(2 r_{0}+r_{1}\right)-2 R \bar{p}\right)^{2}}{2 R \bar{p}}+\frac{\left(\left(r_{1}+2 r_{2}\right)-2 R(1-\bar{p})\right)^{2}}{2 R(1-\bar{p})} \\
& +\frac{\left(\left(2 s_{0}+s_{1}\right)-2 S \bar{p}\right)^{2}}{2 S \bar{p}}+\frac{\left(\left(s_{1}+2 s_{2}\right)-2 S(1-\bar{p})\right)^{2}}{2 S(1-\bar{p})}
\end{aligned}
\]

With some algebra, it can be demonstarted that this expression simplifies to
\[
\begin{equation*}
T_{a}^{2}=\frac{2 N\left\{2 N\left(r_{1}+2 r_{2}\right)-2 R\left(n_{1}+2 n_{2}\right)\right\}^{2}}{(2 R) 2(N-R)\left\{2 N\left(n_{1}+2 n_{2}\right)-\left(n_{1}+2 n_{2}\right)^{2}\right\}} \tag{6.8}
\end{equation*}
\]

Under the null hypothesis that the frequency of alleles is the same in cases and controls (and, as you will see later, that HWE halds in total sample) the test statistic is distributed as \(\chi^{2}\) with one degree of freedom.

An alternative, Armitage's trend test for proportions, can be used to test the null hypothesis. This test is performed using the \(2 \times 3\) genotypic table (6.5). The null hypothesis assumes that the frequency of cases is the same in all genotypic groups; alternative is that the frequency is not the same, but are, however, not totally arbitrary. As it follows from the name of the test, a trend in proportions is assumed, that is the frequency of cases among people with heterozygous genotypes \(A B\) should be exactly between the frequencies of cases in two homozygous classes. This hypothesis may be formalized using parameters beta \(a_{0}\), the expected frequency of cases in the \(A A\) group, and \(\beta_{1}\), the increase in frequency of cases in the \(A B\) group. The expected frequency of cases in the \(A B\) group is then \(\left(\beta_{0}+\beta_{1}\right)\); the frequency of cases in the \(B B\) group is assumed to be ( \(\beta_{0}+2 \cdot \beta_{1}\) ). Parameters \(p_{0}\) and \(p_{1}\) can be estimated using the maximum likelihood; \(\hat{\beta}_{1}=\frac{\left(r_{1}+2 r_{2}\right)-\bar{p} R}{\left(n_{1}+4 n_{2}\right)-\bar{p} N}\), where \(\bar{p}=\frac{2 n_{2}+n_{1}}{2 N}\) is the frequency of the \(B\) allele in the total sample. Then, \(\beta_{0}\) is estimated as \(\hat{\beta}_{0}=\frac{R}{N}-\hat{\beta}_{1} \bar{p}\). Is can be shown that the chi-square test 6.7 based on these expectations takes the form
\[
\begin{equation*}
T_{t}^{2}=\frac{N\left\{N\left(r_{1}+2 r_{2}\right)-R\left(n_{1}+2 n_{2}\right)\right\}^{2}}{R(N-R)\left\{N\left(n_{1}+4 n_{2}\right)-\left(n_{1}+2 n_{2}\right)^{2}\right\}} \tag{6.9}
\end{equation*}
\]

It can be shown that \(T_{t}^{2}=T_{a}^{2}\) if \(n_{1}=2 \sqrt{n_{0} \cdot n_{2}}\), which is equivalent to the condition that HWE halds exactly, in which case the counts of heterozygotes should be \(2 N p q\), with \(p=\sqrt{n_{0} / N}\) and \(q=\sqrt{n_{2} / N}\). When HWE holds for the total sample, the two tests are (at least assymptotically) equivalent. Thus the test \(T_{a}^{2}\) is the test for association in presence of HWE; when HWE does not hold, even in absence of association, the values of \(T_{a}^{2}\) are greater then the values of \(T_{t}^{2}\) test, possibly leading to false positive conclusions about association in presence of deviations from HWE. Therefore the trend test \(T_{t}^{2}\) is to be prefered when testing for genetic association (6).

For study of association between genotype and a quantitative traits, linear regression analysis is performed. Let us denote the vector of phenotypes as \(y\), with particular values \(y_{i}(i=1, \ldots, N)\). Let code the genotypes with a quantitative variable, which reflects the number of \(B\) allels. Thus, we will code \(A A\) as \(0, A B\) as 1 , and \(B B\) as 2 . Let us denote the vector of genotypes as \(g\), with particular values \(g_{i}(i=1, \ldots, N)\) taking the value of 0,1 or 2 . The linear regression model assumes that the expectation of the trait is
\[
E\left[y_{i}\right]=\mu+\beta_{g} \cdot g_{i}
\]
where \(\mu\) is intercept and \(\beta_{g}\) is the coefficient of regression of the genotype on the phenotype.

The estimate of \(\beta_{g}\) is provided by a well-known expresion
\[
\begin{equation*}
\hat{\beta}=\frac{\operatorname{Cov}(y, g)}{\operatorname{Var}(g)}=\frac{\sum x_{i} g_{i} / N-\sum x_{i} \sum g_{i} / N^{2}}{\sum g_{i}^{2} / N-\left(\sum g_{i}\right)^{2} / N^{2}}=\frac{N \sum x_{i} g_{i}-\sum x_{i} \sum g_{i}}{N \sum g_{i}^{2}-\left(\sum g_{i}\right)^{2}} \tag{6.10}
\end{equation*}
\]

Under the null hypothesis, the variance of \(\hat{\beta}\) is \(\operatorname{Var}_{0}\left(\hat{\beta}_{g}\right)=\frac{\operatorname{Var}(y)}{N \cdot \operatorname{Var}(g)}\), and the score test statistic
\[
\begin{equation*}
T_{q}^{2}=\frac{\hat{\beta}_{g}^{2}}{\operatorname{Var}_{0}\left(\hat{\beta}_{g}\right)}=\frac{\operatorname{Cov}(y, g)^{2}}{\operatorname{Var}(g)^{2}} \cdot \frac{N \cdot \operatorname{Var}(g)}{\operatorname{Var}(y)}=N \frac{\operatorname{Cov}(y, g)^{2}}{\operatorname{Var}(y) \cdot \operatorname{Var}(g)} \tag{6.11}
\end{equation*}
\]
is distributed as \(\chi^{2}\) with one degree of freedom. Here, \(\operatorname{Var}_{0} \hat{\beta}_{g}\) denotes the variance under the null hypothesis. Denote the correlation between \(y\) and \(g\), \(\frac{\operatorname{Cov}(y, g)}{\sqrt{\operatorname{Var}(y) \cdot \operatorname{Var}(g)}}\), as \(r\). Then \(T_{q}^{2}=N \cdot r^{2}\).

It is worth to note that Armitage's trend test can be expressed as a linear regression test. In the table 6.5 Let us code the genotypic groups based on the number of \(B\) allels. Thus, we will code \(A A\) as \(0, A B\) as 1 , and \(B B\) as 2 . Let us also code the cases with " 1 " and controls with " 0 ". Let \(y\) denote the vector of phenotypes and \(g\) the vector of genotypes coded in this way. Then, if we perform linear regression of the case-control status onto the number of \(B\) alleles in the genotype, the estimate of the regression coefficient is (following 6.10)
\[
\hat{\beta}=\frac{\sum x_{i} g_{i}-\sum x_{i} \sum g_{i} / N}{\sum g_{i}^{2}-\left(\sum g_{i}\right)^{2} / N}=\frac{\left(r_{1}+2 r_{2}\right)-\bar{p} R}{\left(n_{1}+4 n_{2}\right)-\bar{p} N}
\]

Thus, the expected proportions in the Armitage's trend test are provided by solution of linear regression equation, in which regression of case-control status, coded as " 0 " and " 1 " is performed onto the number of \(B\) alleles. It can be further demonstrated that the trend test statistics 6.9 can be expressed as \(T_{t}^{2}=N \cdot r^{2}\), where \(r^{2}\) is squared coefficient of correlation between \(y\) and \(g\).

In the next two sections we will consider the effects of genetic structure on the standard tests for association described above. The extensive treatment of the problem is mainly due to the seminal works of Devlin, Roeder, Bacanu, and colleagues(7, 8, 9, 10); here, some of their results are repeated.

The genetic structure of the sudy population will be characterized by the kinship matrix - a square matrix, which, for all pairs of individuals in question, provides their pair-wise kinship coefficients (defined at page 94. The kinship coefficient between persons \(i\) and \(j\) will be defined as \(f_{i j}\).

\subsection*{6.2.2 Effects of genetic structure on standard tests}

As it was demonstrated above, the Armitage's trend test for association can be expressed as \(T_{a}^{2}=\frac{\hat{\beta}^{2}}{\operatorname{Var}(\hat{\beta})}\). Genetic structure affects both numerator and denominator of this expression. Indeed, if measurments are dependent, the varaiance of the estimate is likely to be under-estimated if such dependence is not accounted for, leading to the inflation of the tests statistic. Secondly, even in absence of association, the estimate of the effect may be biased, again, leading to increased value of the test statistic.

Let us consider following artificial example. Let study cases are closely related individuals - sibs \(\mathbb{1}^{1}\) - coming from one family, and study contols are sibs from other family. In essence, we compare the squared frequency difference between the two groups to the variance of this difference. If groups are formed by independent individuals, we can detect arbitrary small frequency differences by increasing the sample size, and consequently, decreasing the variance of the frequency estimate. This definitely is not the case when our study sample consist of a large sibship - whatever is the sibship size, only four alleles may be present in the parents, and the precision of the estimate of the allele frequency in the general population is limited - as if we had two people at maximum. If we do not take this consideration into account, we are likely to over-estimate the precision

\footnotetext{
\({ }^{1}\) brothers and sisters
}
of the frequency estimate; the denominator of the test statistic becomes small, and the statistic becomes large. Secondly, the parents of the "case" and "control" sibships have a high chance to be genotypically different, just by chance; and any genotypic configuration leading to the different number of \(B\) alleles will be reflected in diffence in frequencies between sibships, given the sibships are large. Assuming HWE, the chance that two parental couples will have different number of \(B\) alleles is one minus the chance that they will have the same number of \(B\) alleles:
\[
\begin{aligned}
P\left(\# B_{1} \neq \# B_{2}\right) & =1-\left(P\left(\# B_{1}=\# B_{2}=0\right)+P\left(\# B_{1}=\# B_{2}=1\right)+\ldots+P\left(\# B_{1}=\# B_{2}=4\right)\right) \\
& =1-\left(P\left(\# B_{1}=0\right) P\left(\# B_{2}=0\right)+\ldots+P\left(\# B_{1}=4\right) P\left(\# B_{2}=4\right)\right) \\
& =1-\left(q^{8}+16 p^{2} q^{6}+36 p^{4} q^{4}+16 p^{6} q^{2}+p^{8}\right)
\end{aligned}
\]
which, for a common allele with frequency of 0.2 translates to the probability 0.64 .

Let us quantify these two sources of bias. Following (9) let us assume that the number of cases and the number of controls is the same, \(N\). Let us denote the vector of genotypes, coded as 0,1 , and 2 of cases as \(X\), with \(X_{i}(i=1, \ldots, N)\) being the genotype of the \(i\)-th case, and the vector of genotypes of controls as \(Y\) (with \(Y_{j}, j=1, \ldots, N\) being the genotype of \(j\)-th control). The trend and the allelic tests statistic are proportional to the square of \(T=\sum X_{i}-\sum Y_{j}\). The variance of this statistic, in general form, is
\[
\begin{align*}
\operatorname{Var}(T)= & \sum_{i=1}^{N} \operatorname{Var}\left(X_{i}\right)+\sum_{i=1}^{N} \operatorname{Var}\left(Y_{i}\right)  \tag{6.12}\\
& +2 \sum_{i<j} \operatorname{Cov}\left(X_{i}, X_{j}\right)+2 \sum_{i<j} \operatorname{Cov}\left(Y_{i}, Y_{j}\right) \\
& -2 \sum_{i} \sum_{j} \operatorname{Cov}\left(X_{i}, Y_{j}\right)
\end{align*}
\]

Let us consider a sitatuation in which cases come from one popualtion, and controls from the other population; each of the populations is in HWE and the difference between the populations is characterized with \(F_{s t}\) (see equation 6.6). Under this model, \(\operatorname{Var}\left(X_{i}\right)=\operatorname{Var}\left(Y_{i}\right)=2 p q\left(1+F_{s t}\right)\), and the covariance between any pair of genotypes from the same population is \(4 p q F_{s t}\). Then
\[
\begin{align*}
\operatorname{Var}(T)= & 2 p q\left(1+F_{s t}\right) \cdot N+2 p q\left(1+F_{s t}\right) \cdot N \\
& +2 \cdot 2 p q F_{s t} \cdot N(N-1) / 2+2 \cdot 2 p q F_{s t} \cdot N(N-1) / 2+0 \\
= & 4 p q N \cdot\left(1+F_{s t}+2 F_{s t}(N-1)\right) \approx 4 p q N\left(1+2 N F_{s t}\right) \tag{6.13}
\end{align*}
\]
here, \(4 p q N\) corresponds to the binomial variance of \(T\) in absence of genetic structure, while \(F_{s t}+2 \cdot F_{s t} \cdot(N-1)\) reflects the inflation of the variance. As the second term is the function of the sample size, large inflation may be achieved even with small vaules of \(F_{s t}\). Note that here the sample size is \(2 N\), as we assumed \(N\) cases and \(N\) controls.

Above we have considered an example in which we know the \(F_{s t}\) between the population of cases and the population of controls. In a practical study, a number of cases and controls is usually sampled from each genetically different population. Let the proportion of individuals sampled from population \(c\) among cases is \(a_{c}\) and the proportion of individuals coming from that population among controls is \(u_{c}\). Then, it can be shown that
\[
\begin{equation*}
\operatorname{Var}(T) \approx 4 p q N\left(1+2 \cdot N \cdot F_{s t} \cdot \sum_{c}\left(a_{c}-u_{c}\right)^{2}\right) \tag{6.14}
\end{equation*}
\]

As it follows from this equation, the variance of the estimated frequency difference depends on the composition of the sample. Maximal inflation is achieved when the cases and controls are sampled from different populations, while if \(\sum_{c}\left(a_{c}-u_{c}\right)^{2}=0-\) which is achieved by sampling equal number of cases and controls from each sub-population - the variance inflation is minimal.

These results canbe generalized to arbitrary relations between cases and controls. Let us denote kinship between cases \(i\) and \(j\) as \(f_{i j}^{X}\), kinship between controls as \(f_{i j}^{Y}\), and the kinship between a case and a control as \(f_{i j}^{X Y}\). Then(9)
\[
\begin{align*}
\operatorname{Var}(T)=4 p q N \cdot\left(\begin{array}{ll}
1 & +F_{s t}+\frac{2}{N} \sum_{i<j} f_{i j}^{X} \\
& +\frac{2}{N} \sum_{i<j} f_{i j}^{Y}-\frac{2}{N} \sum_{i} \sum_{j} f_{i j}^{X Y}
\end{array}\right) \tag{6.15}
\end{align*}
\]

This equation demonstrates that the variance is inflated not only in a case when cases and controls come from genetically distinct populations, but that also "criptic relations" among either cases (some \(f_{i j}^{X}>0\) ) or controls (some \(f_{i j}^{Y}>0\) ) may lead to increased variance of the test. Because summation (e.g. \(\left.\sum_{i<j} f_{i j}^{X}\right)\) is performed overl all pairs of cases/controls, cryptic relations may have a strong impact onto inflation of the variance.

Generally, the variance of the \(T\) can be expressed as \(\operatorname{Var}(T)=4 N p q\left(1+F_{s t}+\right.\) \(\left.D\left(f^{X}, f^{Y}, f^{X Y}\right)\right)\). Here, \(4 N p q\) corresponds to the binomial variance assuming HWE and independence between cases and controls, the second term \(-F_{s t}-\) accounts for increase in variance due to deviation from HWE, and the last, which is a function of kinship \(-D\left(f^{X}, f^{Y}, f^{X Y}\right)\) - accounts for dependencies bewteen cases and controls. The test \(\frac{T^{2}}{4 N p q\left(1+F_{s t}+D\left(f^{X}, f^{Y}, f^{X Y}\right)\right)}\) is distributed as \(\chi^{2}\) with one degree fo freedom.

The allelic and Armitage's trend statistics can be expressed as \(T_{a}^{2}=\frac{T^{2}}{4 N p q}\) and \(T_{t}^{2}=\frac{T^{2}}{4 N q q \cdot\left(1+F_{s t}\right)}\). What is their distribution when there is relatedness / stratification bewteen cases and controls? Clearly, both of them are distributed as \(\tau^{2} \cdot \chi^{2}\), where \(\tau^{2}>1\) is the variance inflatiopn factor. It easy to show that it is equal to \(\tau^{2}=1+F_{s t}+D\left(f^{X}, f^{Y}, f^{X Y}\right)\) for the allelic and \(\tau^{2}=\) \(\frac{1+F_{s t}+D\left(f^{X}, f^{Y}, f^{X Y}\right)}{1+F_{s t}}\) for the trend test.

In above, we have accounted for the over-dispersion of the test statistic due to possible relatedness between the cases and controls, and demonstrated that the allelic and the trend test are inflated by some constant \(\tau^{2}\), which reflects the inflation of the variance, \(\operatorname{Var}(T)=\sigma^{2} \cdot \tau^{2}\), where \(\sigma^{2}\) is the variance estimated under the assumption of independence. However, in the beginning of this section we have discussed another source of the infaltion of the test statistic - the bias in frequencies between cases and controls due to confounding. Let cases and controls are sampled from different populations, whose difference is characterized by \(F_{s t}\), and let for some marker the allele frequency difference - which occures entirely due to genetic structure of the sampels - between casese and controls is \(d\). The test statistic \(T\) is then distributed as Normal with mean \(N d\) and variance computed in previous section, \(\sigma^{2} \cdot \tau^{2}, N\left(N d, \sigma^{2} \tau^{2}\right)\). To address the issue of bias, we need to figure out the distribution of the mean \(-2 N d\). In general, the expected frequency difference is zero - indeed, if we consider a large number of subpopulations, or a large number of markers, if allelic frequencies are determined by random effects, the deviation is likely to occure in any direction, resultin in zero on average. The variance of the \(d\) is
\(2 p q F_{s t}\), thus the variance of \(2 N d\) is \((2 N)^{2} 2 p q F_{s t}{ }^{2}\) Thus finally, \(T\) is expected to be distributed as \(N\left(0, \sigma^{2} \cdot \tau^{2}+8 p q F_{s t} N^{2}\right)\). If we consider the variance of allelic test, \(\sigma^{2}=4 p q N\), then \(T \sim N\left(0, \sigma^{2}\left(\tau^{2}+2 F_{s t} N\right)\right)\), and, denoting \(2 F_{s t} N\) as \(\nu^{2}\), \(T \sim N\left(0, \sigma^{2}\left(\tau^{2}+\nu^{2}\right)\right)\). Similar expression may be obtained for the Armitage's trend test.

Thus, in presence of genetic structure both allelic and the trend test are distributed as \(\left(\tau^{2}+\nu^{2}\right) \cdot \chi^{2}\). Note that while \(\sigma^{2}\) is a function of allelic frequency, the inflation factors \(\tau^{2}\) and \(\nu^{2}\) depend on the differentiation between the populations in question, as measured by \(F_{s t}\), sample size, \(N\), and the composition of the case-control sample \(\left(a_{c}, u_{c}\right)\), but does not depend on the allele frequency.

As it was mentioned earlier, the Armitage's trend test can be re-formulated as a regression-based score test, in which genotypes are coded as 0,1 , and 2 , and phenotypes as 0 and 1 . Therefore all above arguments apply to the analysis of quantitative traits as well. The test statistic \(T_{q}^{2}=\frac{\hat{\beta}_{g}^{2}}{\operatorname{Var}\left(\hat{\beta}_{g}\right)}\) in which \(\hat{\beta}\) and \(\operatorname{Var}(\hat{\beta})\) are estimated using independence assumption is inflated because of overdispersion \((\operatorname{Var}(\hat{\beta})\) is under-estimated if the relatedness is not accounted for), and also \(\hat{\beta}_{g}\) may be biased, because different genetic populations may well be characterized by different mean value of the trait (e.g. because of environmental influences). It can be shown that also for quantitative traits \(T_{q}^{2}\) is distributed as \(\left(\tau^{2}+\nu^{2}\right) \cdot \chi^{2}\), where \(\tau^{2}\) is inflation due to over-dispersion, and \(\nu^{2}\) is the inflation because of bias.

\subsection*{6.2.3 Genomic control}

From previous section it follows that in presence of genetic structure the standard association statistics may be inflated and the distribution is described as \(\left(\tau^{2}+\nu^{2}\right) \cdot \chi^{2}\). Let us denote \(\left(\tau^{2}+\nu^{2}\right)\) as \(\lambda\) for short. \(\lambda\) depends on the genetic structure of the sample, as characterized by pairwise kinship, and sample size, \(N\). For binary trait analysis, it also depends on the composition of the case-control sample, as expressed by the proportio of cases/conrols coming from particular population, \(c\left(a_{c}, u_{c}\right)\). For quantitative trait, it depends of neritability of the trait, and environmentally detemined differences co-occuring with the difference in kinship. However, \(\lambda\) does not depend on the allele frequency. Therefore, for any particular study sample, if \(F_{s t}\) is constant over the genome, \(\lambda\) is also a constant.

Therefore \(\lambda\) can be estimated from the genomic data, using "null" loci - a set of random markers, which are believed not to be associated with the trait. This estimate can then be used to correct the values of the test statistic at the tested loci - a procedure, known as "genomic control". The test statistics computed from these loci thus estimates the distrbution of the test statistic under the null hypothesis of no association. Let us consider \(M\) "null" markers, and denote the test statistic obtained from \(i\)-th marker as \(T_{i}^{2}\). Given genetic structure determines inflation \(\lambda, T^{2} \sim \lambda \cdot \chi_{1}^{2}\). The mean of a random variable coming from \(\chi_{1}^{2}\) is equal to 1 ; if a random variable comes from \(\lambda \cdot \chi_{1}^{2}\), the mean would be \(\lambda\). Thus we can estimate \(\lambda\) as the mean of the obtained "null" tests. In practice, it is recommended to use the ratio between the observed median

\footnotetext{
\({ }^{2}\) For the case when some proportion, \(a_{c}\), among cases comes from population \(c\), and some proportion of controls, \(u_{c}\), comes from population \(c \operatorname{Var}(d)=2 p q F_{s t} \sum_{c}\left(a_{c}-u_{c}\right)^{2}\)
}
and the one expected for \(\chi_{1}^{2}\) :
\[
\begin{equation*}
\hat{\lambda}=\frac{\operatorname{Median}\left(T_{i}^{2}\right)}{0.4549} \tag{6.16}
\end{equation*}
\]

For the tested markers, the corrected value of the test statistic is obtained by simple division of the original test statistic value on \(\hat{\lambda}, T_{\text {corrected }}^{2}=T_{\text {original }}^{2} / \lambda\). Note that this procedure is correct only if the same number of study participants was typed for any marker, as you will see later.

As you can see, the genomic control procedure is computationally extremely simple - one needs to compute the test statistic using a simple test (e.g. score test), compute the median to estimate \(\lambda\), and divide the original test statistics values onto \(\hat{\lambda}\).

How to choose "null loci" is a GWA study? In a genome, we expect that a small proportion of markers is truly associated with the trait. Therefore in practice, all loci are used to estimate \(\lambda\). Of cause, if very strong (or multiple weak) true associations are present, true association will increase the average value of the test, and genomic control correction will be conservative. To relax this, it has been suggested(11) to use, say, \(95 \%\) of the least significant associations for estimation of the inflation factor; however, the question of selection of cut-off is not obvious, and all markers are used in practice in most studies.

We have observed that the inflation factor \(\lambda\) is a function of sample size, \(N\) - the bigger is the sample size, the larger is inflation. If this is the case, how can we compare inflations between different GWA studies? A good idea is to use a standardized inflation, say, inflation per 1,000 subjects. For a quantitative trait analysis, inflation factor can be expressed as \(\lambda \approx(1+N * D(\) sample \())\), where \(N * D\) (sample) is a term, which grows linear with sample size, at some rate determined by sample characteristics.

Therefore a standarsized inflation factor can be etimated as
\[
\hat{\lambda}_{1000}=1+\frac{\hat{\lambda}-1}{N} \cdot 1000
\]
where \(N\) is the sample size and \(\hat{\lambda}\) is the estimate from the total sample.
What about a case-control sample? For such samples, a standardized \(\lambda\) is computed for a fixed number of cases and controls, say 1000; we will denote such standardized inflation as \(\lambda_{1000,1000}\) to distinguish it from the one computed for a quantitative traits. If we denote the number of cases as \(N_{a}\) and the number of controls as \(N_{u}\), then
\[
\hat{\lambda}_{1000,1000}=1+\frac{500 \cdot(\hat{\lambda}-1) \cdot\left(N_{a}+N_{u}\right)}{N_{a} \cdot N_{u}}
\]

Clearly, we have assumed that in our GWA study the sample size was equal for all studied markers; this fact allows for very simple estimation of \(\hat{\lambda}\) (equation 6.16) and correction of the test statistic. It may happen, however, that the number of participants typed for different marker loci is different; for example, this may happen when all study participans were typed for SNP panel one, and then a part of the study was additionally typed at a different panel. In such situation, expectation of a \(T_{q}^{2}\) test for a quantitative trait can be expressed as
\[
E\left[T_{q}^{2}\right]=1+\left(\lambda_{1000}-1\right) \cdot \frac{N}{1000}
\]

It is straightforward to obtain an estimate of \(\lambda_{1000}\) by performing linear regression of the observed test statistic values onto the sample size used; note that the intercept should be fixed to 1 in this procedure. Other, more effective procedures may be thought of.

For binary traits, if sample is composed from \(N_{a}\) cases and \(N_{u}\) controls, the expression for the expected value of the test statistic is similar to that obtained for quantitative traits, however, geometric mean of the number of cases and controls are used(12):
\[
E\left[T_{t}^{2}\right]=1+\left(\lambda_{1000,1000}-1\right) \cdot \frac{\frac{1}{1000}+\frac{1}{1000}}{\frac{1}{N_{a}}+\frac{1}{N_{u}}}
\]

This relation can be used to estimate \(\lambda_{1000,1000}\).
We have considered the genomic control procedure for the tests assuming additive effects of the locus onto phenotype. Under this model, inflation of the tests \((\lambda)\) does not depend on allelic frequency, which allows a straightforward estimation of \(\lambda\) and further correction of the tests. Does the same apply to other genetic models?

In figure 6.4 the results for additive, dominant, recessive, and over-dominant model tests are presented. Simulated study consisted of 1000 cases and 1000 controls; the differentiation between the case and control populations was assumed \(F_{s t}=0.002\). Armitage's trend test was used in analysis. One can see that when a common polymorphism is studies \((0.05<q<0.95)\), the additive model test (green line) does not depend on the frequency of the \(B\). The range of allele frequencies in which the additive model does not depend on these depends of the values of \(F_{s t}\) and the sample size: the larger differentiation, and the smaller the sample size, the narrower is the range.

However, other one degree of freedom tests (recessive, red; dominant, blue; over-dominant, cyan) do depend on the allele frequency very much across all the range of allelic frequencies (figure 6.4 see (13) for details).

Thus, it is important to remember that the genomic control was developed for, and works with additive genetic models; even in this framework, behaviour of \(\lambda\) depends on allelic frequencies when theses are too low or too high - thus application of genome-wide derived inflation factor to such marker loci may be incorrect for loci with low minor allele frequency. What is somewaht sustaining statistically is the fact that such correction will be conservative, and not liberal - but the same may be worrysome biologically (e.g. missed true positives). For other types of model, lambda can, at least in theory, be estimated taking into account the allelic frequency of the locus in question. However, these methods are not implemented yet in packages for genome-wide association analyses.

Other important thing to remember about genomic control is that it assumes uniorm \(F_{s t}\) across the genome. This may not be the case for some (e.g. selected) genomic regions. Such regions may still generate very high test statistic, even after appropritate genomic control correction(14); basically, the frequency distribution at such loci may vary from population to population much more then the average across the genome. A number of methods taking genetic structure of study population into account directly (structured association, EIGENSTRAT, reviewed later), can deal with such situations.

Finally, what levels of genomic control inflation parameter are acceptable in GWA studies? As the question is of how much the total test statistic is affected,


Figure 6.4: Mean \(T_{t}^{2}\) test statistic as frequency of the \(B\) allele, \(q\). Models considered: additive, green; recessive, red; dominant, blue; over-dominant, cyan.
non-standardized inflation may be used to address the question. Despite of lack of clear guidance, the general practice in the field is to consider the values of \(\hat{\lambda}<1.01\) as small, \(\hat{\lambda}<1.05\) as moderate and still acceptable. If \(\hat{\lambda}>1.1\), this suggest strong influence of genetic structure or other design factors on the test statistic. While GC statistically correct method, such analysis lacks power; good practice is to consider use of other methods, which take the structure of the sample into account in diect manner.

\subsection*{6.3 Analysis of structured populations}

A study populaion may be structured in at least two ways. Firstly, while study participants may be "independent" in that kinship between them is expected to be very low (e.g. random sample from a large outbred population), they may belong to rather different genetic populations. An extreme example of such study is a study aiming to combine the data from a sample obtained by random ascertainment from the population of Bejing and a sample from Amsterdam. While in any of the samples people are only remotedly related ("independent"), the two samples in question are characterized by high genetic differences; also the environmental influences may be very different for the two populations. The distribution of the trait under the study may be essentially different between the two populations: not only the mean values, but also variances.

On the other hand, consider a family-based sample from genetically isolated population. Here, environmental influences are more or less uniform for all study participants; the distribution of the trait may be assumed to be governed by the same set of rules for all study participants. However, they are characterized by high and variable kinship between each other.

These two flavours of stratification may be expressed in terms of kinship. When we talk about different "populations", we assume very low kinship and high genetic differentiation between the members of these; when we talk about "relatedness", we assume high kinship and small degree of differentiation. This is illustrated at figure 6.5 Roughly speaking, when we expect that study participants share common ancestors few (1-4) generation ago, the kinship between study participants is high, and the study can be classified as a family-based one (Study 1 of figure 6.5). When common ancestors between study participants is expected say \(>5\) generations ago, the kinship between study participants is low, and such study may be classified as a population-based sample of "independent" people (Studies 2 and 3 of figure 6.5). Now consider two groups of study participants (samples 2 and 3 ). Any person from group 2 is expected to share a common ancestor with a person from the same study group with much higher probability than with a person form group 3; thus these represent two genetic population (see page 95 for the retrospective definition of genetic population). It shouldbe kept in mind that if kinship is very low (expected common ancestors dozns of generations ago), this would translate in high degree of genetic differentiation. Moreover, this reflects a long history of isolation, which usually means geographic sepration, and accumulation of cultural and other environmentsl differences between the populations, which may be crucial in association studies.

Of cause, a particular study is usually characterized by some mixture of populations and some relatednes between study participants. We will, however,


Figure 6.5: Three samples from two populations. 1: Family-based sample; 2,3: random sample of "independent" people
first consider the two extreme scenarios - analysis of samples of "independent" subjects from different populations, and analysis of a family-based study.

\subsection*{6.3.1 Structured association}

If study populayion consist of a mixture of several distinct genetic populayions, and, within each sub-population, the study participants are remotedly related, structured asoiation analysis may be the method of choice. In such analysis, effect and its variance are estimated within each strata separately, and then these estimates are pooled to generate global statistics. The strata can be known from design (e.g. place of birth or ethnicity of parents) or estimated from GWA data. By doing this, we allow for arbitrary trait distribution, characterized by stratum-specific mean and variance, in each stratum. Clearly, this may be crucial when different populations characterized by different environments are included in analysis.

Combining the evidence across strata may be done using a number of methods, e.g. Cochran-Mantel-Haenszel test for binary outcomes. One of the most simple ways to combine the evidences coming from multiple strata - or studies - is to use fixed effects inverse variance meta-analysis.

In essence, this method is equivalent to combining likelihoods coming from separate studies, using quadratic approximation. Denote coefficients of regression estimated in \(N\) studies/strata as \(\beta_{i}\), and associated squared standard errors of the estimates as \(s_{i}^{2}\) where \(i \in 1,2, \ldots, N\). Note that the regression coefficient should be reported on the same scale, e.g. centimeters, meters, or using observations reported on the standard normal scale. Define weights for individual
studies as
\[
w_{i}=\frac{1}{s_{i}^{2}}
\]

Then the pooled estimate of the regression coefficient is
\[
\beta=\frac{\sum_{i=1}^{N} w_{i} \beta_{i}}{\sum_{i=1}^{N} w_{i}}
\]

As you can see, the weights have straightforward interpretation: the bigger the weight of the study (meaning the small is the standard error in the study), the larger is the contribution from this study onto the pooled estimate.

The standard error of the pooled estimate is computed as
\[
s^{2}=\frac{1}{\sum_{i=1}^{N} w_{i}}
\]
and the \(\chi^{2}\)-test for association is computed in standard manner as
\[
T^{2}=\frac{\beta^{2}}{s^{2}}=\frac{\left(\sum_{i=1}^{N} w_{i} \beta_{i}\right)^{2}}{\sum_{i=1}^{N} w_{i}}
\]
or, alternatively, the Z-test is
\[
Z=\frac{\beta}{s}=\frac{\sum_{i=1}^{N} w_{i} \beta_{i}}{\sqrt{\sum_{i=1}^{N} w_{i}}}
\]

When binary traits are studied, and results are expressed as Odds Ratios with \(P\)-values, it is also possible to apply inverse variance method. For this, you need to transform your Odds Ratios using natural logarithm, and, on this scale, estimate the standard error. Generic inverse variance pooling may be applied to the data transformed this way; the final results are back-transformed onto Odds Ratio scale using exponentiation.

In meta-analysis procedure, it is assumed that study populations are composed of genetically homogenous "unrelated" individuals. Each study is first analysed seprately, and genomic control should either show no inflation of the test statistic, or a small one, which should be corrected with GC prior to combinaning of strata/studies. If some of the stata demonstrate large residual inflation, this may suggest further sub-structure present in that strata - it should be dealt with using further sub-division and structure association, or methods described further in this chapter, prior to attempting the pooling of results.

The meta-analytic methodology to perform structured association analysis outlined above has advantage of simplicity. However, it assumes goodness of quadratic approximation of the likelihood function, which is achieved if numbers are large. If numbers are not large (e.g. rare polymorphisms), a more complicated strategy can be used. A regression model, allowing for study-specific mean, effects of nuisance parameters, and residual variances, but unique (across the studies) effect of interest may be formulated. Analysis of such model is equivalent to the above-described meta-analysis procedure, under large numbers assumption.

\subsection*{6.3.2 Mixed models based approach}

In this section we will consider the methods used to analyse samples in which close family relations may be persent; theses may be family-based samples from outbred populations, or random samples from genetically siaolted populations, where, due to limited population size, relatives are likely to be present by chance.

The mixed model based ideology stems from the classical animal breeding and human heritability analysis methodology, dating back to works of Fisher(15). He proposed a model in which a very large number of Mendeian genes contribute small effects to the phenotype (infenitisimal model). We may express this model as
\[
\begin{equation*}
y_{i}=\mu+G_{i}+e_{i} \tag{6.17}
\end{equation*}
\]
- the value of the trait of \(i\)-th person is sum of grand population mean, contribution from multiple additively acting genes of small effect \(\left(G_{i}\right)\), and residual error \(\left(e_{i}\right)\).

Under this assumption, the distribution of the trait in a pedigree is described by a multivariate normal distribution with number of dimensions equal to the number of phenotyped people, with the expectation of the trait value for some individual \(i\) equal to
\[
E\left[y_{i}\right]=\mu
\]
where \(\mu\) is grand population mean (intercept). The variance-covariance matrix is defined through its elements \(V_{i j}\) - covariance between the phenotypes of person \(i\) and person \(j\) :
\[
V_{i j}= \begin{cases}\sigma_{e}^{2}+\sigma_{G}^{2} & \text { if } i=j  \tag{6.18}\\ 2 \cdot f_{i j} \cdot \sigma_{G}^{2} & \text { if } i \neq j\end{cases}
\]
where \(\sigma_{G}^{2}\) is variance due genes, \(f_{i j}\) is kinship between persons \(i\) and \(j\), and \(\sigma_{e}^{2}\) is the residual variance. The proportion of variance explainable by the additive genetic effects,
\[
h^{2}=\frac{\sigma_{G}^{2}}{\sigma_{G}^{2}+\sigma_{e}^{2}}
\]
is termed (narrow-sense) "heritability".
Fixed effects of some factor, e.g. SNP, may be included into the model by modifying the expression for the expectation, e.g.
\[
E\left[y_{i}\right]=\mu+\beta_{g} \cdot g
\]
leading to so-called "measured genotypes" model(16).
In essence, this is a linear mixed effects model - the one containing both fixed (e.g. SNP) and random (polygenic) effects. A large body of literature is dedicated to this class of models, and finding the solution for such regression represents no great methodological challenge. Applicability of this model to GWA analysis was first proposed by Yu and colleagues(17). Two problems occure with application of this model to GWA analysis: firstly, if standard Maximum Likelihood (or Restricted Maximum Lileihood) approach is used to estimate this model, the computational time is excessive(18); secondly, the pedigree and thus the kinship matrix - may be not known for study participant.

Let us first consider exisiting solutions to the first problem. High computational complexity occures if both random and fixed effects part of the model are estimated simulatneously, and this procedure is repeated for every SNP
analysed. However, we may assume that the effects of SNP are liklely to be small. Thus, inclusion of the SNP into the model is not likely to change the estimates of the components of variance. Therefore a two-step procedure was suggested(19; 20): firstly, the mixed model containing all terms but these involving SNPs, is estimated (this includes estimation of the components of variance, and possibly the nuisance fixed effects); secondly, these estimates are then used to compute a test for association for every SNP in analysis. For details on this test, see (19) for original formulation, and (20) for extension to multiple variables and interaction effects tested in step two. The model is implemented in MACH and ProbABEL software packages.

Another method, bearing close resemblance to the method described above, is GRAMMAR - Genome-wide Rapid Analysis using Mixed Models and Regression(18). Here, in step one, not only the mixed model is estimated, but also the environmental residuals,
\[
\hat{e}_{i}=y_{i}-\left(\hat{\mu}+\hat{G}_{i}\right)
\]
are estimated. These are free of correlations, and can be used as the new "trait" in further analysis. This procedue allows for very fast GWA analysis in step 2 (for large sample sizes, much faster then above-described two-step approach), also it has a great flexibility as the environmental residuals can be analysed using a large variaty of methods. At the same time, the GRAMMAR test for association is conservative, and the estimates of the effects obtained in step 2 are downward biased(18). While conservativity of the test may be dealt with using "reverse genomic control"(21), the issue of bias in effect estimates has no solution yet. The GRAMMAR model is implemented in GenABEL software package(22).

Strictly speaking, above described two-step tests are correct if the distributions of covariates in the first and the second parts of the model are independent conditional on the estimated phenotypic variance-covariance matrix. This assumption is most likely to be true when the covariates included in the base model are environmental ones, and thus are not expected to exhibit conditional correlation with SNPs. However, when endogenous risk factors, such as e.g. body mass index, are included as the covariates in the base model, some SNPs are expected to exhibit covariance with this covariate. In such situation the two-step test is not strictly correct, but given relatively weak SNP-phenotype correlations normally observed in GWAS should normally keep good statistical properties in most situations. The properties of the test under such conditions are still awaiting their description.

The methodology described here applies to study of association of quantitative traits. For binary traits, no formal practical GWA solution is available yet. The best current strategy to analyze binary traits in samples of relatives may be to treat the binary outcome as if it was quantitative. As you have seen with Armitage's trend test, which is equaivalent to the score test for a quantitative variable when oucome is coded with "0" and " 1 " (page 106), this approach leads to correct inferences about significance.

\subsection*{6.3.3 Estimation of kinship matrix from genomic data}

Another problem with application of mixed models to analysis of GWA data is that pedigree is required to estimate the kinship matrix. The expectation
of kinship can be estimated from pedigree data using standard methods, for example the kinship for two outbred sibs is \(1 / 4\), for grandchild-grandparent is \(1 / 8\), etc. For an outbred person, the kinship coefficient is \(1 / 2-\) that is two gametes sampled from this person at random are IBD only if the same gamete is sampled. However, in many situations, pedigree information may be absent, incomplete, or not reliable. Moreover, the estimates obtained using pedigree data reflect the expectation of kinship, while the true realization of kinship may vary around this expectation. In presence of genomic data it may therefore be desirable to estimate the kinship coefficient from these, and not from pedigree. It can be demonstrated that unbiased and positive semi-definite estimator of the kinship matrix (23) can be obtained by computing the kinship coefficients between individuals \(i\) and \(j\) with
\[
\begin{equation*}
\hat{f}_{i j}=\frac{1}{L} \sum_{l=1}^{L} \frac{\left(g_{l, i}-p_{l}\right)\left(g_{l, j}-p_{l}\right)}{p_{l}\left(1-p_{l}\right)} \tag{6.19}
\end{equation*}
\]
where \(L\) is the number of loci, \(p_{l}\) is the allelic frequency at \(l\)-th locus and \(g_{l, j}\) is the genotype of \(j\)-th person at the \(l\)-th locus, coded as \(0,1 / 2\), and 1 , corresponding to the homozygous, heterozygous, and other type of homozygous genotype.(21; 23, 24) The frequency is computed for the allele which, when homozygous, corresponds to the genotype coded as " 1 '.

Inetrestingly, use of the kinship matrix instead of pedigree kinship (when available) may lead to higher power, especially when "dense" pedigres and traits with high heritability are considered (YSA, unpublished data). This is likely to happen because genomic-based kinship is likely to reflect true genetic relations better than (possibly not completely correct) pedigree expectations.

\subsection*{6.3.4 EIGENSTRAT and related methods}

EIGENSTRAT(24) method combines the ideas underlying structured association and the genomic kinship matrix. Basically, it allows for structured association, with strata and more subtle structure identified through the use of genomic data.

The genomic kinship matrix is computed as described earlier (equation 6.19). This matrix reflects genetic similarities between study participants. A reverse metric, distance matrix, is computed as \(0.5-f\). Classical multi-dimentional scaling (CMDS) is applied to identify a number (say, \(k\) ) of first principal components (PCs) of variation of the distance matrix. The idea behind CMDS is mapping of the all pair-wise distances defined by the original matrix to some \(k\)-dimentional space. Each study participant is presented as a dot in this space, and these dots are constructed in such way, that the distance between these is maximally close to the distances observed in the original distance matrix. Thus, if two genetically distinct populations are present in a sample, the first principal axe of variation \((k=1)\) will identify these. Generally, of \(n\) distinct populations are present, these can be identified by \(k=n-1\) first axes.

To demonstrate this principle, results of CMDS of the genomic distance matrix for the HapMap participants, and a number of people from Europe, are presented at figure6.6. On panel \(\mathbf{A}\), the results of CMDS of the distance matrix to to the space of the first two PCs are present. Inter-continental differences


Figure 6.6: The European genetic structure (based on 273,464 SNPs). Three levels of structure as revealed by PC analysis are shown: A) inter-continental; B) intra-continental; and C) inside a single country (Estonia), where median values of the PC1\&2 are shown. D) European map illustrating the origin of sample and population size. CEU - Utah residents with ancestry from Northern and Western Europe, CHB - Han Chinese from Beijing, JPT - Japanese from Tokyo, and YRI - Yoruba from Ibadan, Nigeria. (reproduced from (5)) NEED PERMISSION FROM PLoS ONE!
are obvious - there are three distinct clusters, corresponding to Yoruba (yellowbrown), Asian (green and dark brown), and European (blue and red) study participant. You can see that mapping to the first two principal components perfectly distinguishes these three populations. The same methodology can be used to distinguish between more subtly divided populations - at panel \(\mathbf{B}\) of figure 6.6 the mapping of European populations is present. The panel \(\mathbf{C}\) presents even further detalisation of genetic distances - to the level of single country.

Note that configuration of European cluster in panel \(\mathbf{A}\) is not exactly the same as in panel \(\mathbf{B}\). This happens because the distance matrix, computed based on formula 6.19, depends on allelic frequencies, as estimated in the total sample. Clearly, these are different, when all continents or Europeans only are used. Consequtive analysis of sub-samples allows distinguishing more and more subtle differences between populations.

Interestingly, the genetic distances co-incide very well with geographic distances, i.e. the populations close geographically are also close genetically. Thus, most isolation between human populations is explained by geographic distance.

How these principal components may be incorporated in association anal-
ysis? Different populations may have diferent mean values (or prevalences) of the trait under the study, which, coupled with genetic differences, leads to confounding and false positives in association study. A simple method would be to try to account for these differences in the means by incorporating the principal components as covariates into association analysis(11). Another, related method would be to perform linear adjustment of both the phenotype and the genotype onto PCs (i.e. compute the residuals from linear regression of genotypes/phenotypes onto PCs). Then simple score test for association ( \(T_{q}^{2}\), equation 6.11) can be used to analyze association. This comprises the original EIGENSTRAT methodology suggested by Price and colleagues(24).

A practically interesting question is how many PCs to include into analysis? By default, EIGENSTRAT uses 10 PCs, but this is rather arbitrary number. In general, "significant" PCs should be included into analysis. Patterson and colleagues(25) suggested including the leading PCs, which significantly explain progressively more genetic variation. Alternatively, it was argued that PCs significantly associated with the phenotype should be included(26), as inclusion of other PCs is not only unnecessary, but may also reduce power.

Note that methods described here allow for differences in means between the populations, but not for the difference in variances. It would be interesting to address the question what effects violation of this assumption may have on association studies, and develop extended methods allowing for difference in variances. This would first probably require a study of inter-populatinal variances.

\subsection*{6.3.5 Summary: what method to use?}

What methods should be used for analysis of particular study? An simplified overview is provided at figure 6.7 .

Basically, if study participants come from the same genetic population, are characterozed by low degree of kinship, and confounding, as measured with genomic control (GC) inflation parameter \(\lambda\) is small, GC is enough to correct for residual bias.

If study participants come from substantially different populations, and, for each population, above conditions hold, structured association may be used.

If study participants are characterized by high degree of kinship, mixedmodel based methods should be used.

EIGENSTRAT and PC-based adjustment methods should be used in somewhat intermediate situation when differentiation between the populations and kinship between study participants is not too high: while substantial population differentiation may mean differences in variances, which EIGENSTRAT/PCBA does not account for, it is also known that these methods do not perform very well in pedigrees (YSA, Najaf Amin, unpublished data).

Clearly, a particular study may have its own specifics and should be considered separately. As difference between outlined scenarios is quantitative rather than qualitative, there is no single recipe for analysis when genetic structure is present in a study sample.


Figure 6.7: Applicability of different methods for association analysis.

\subsection*{6.4 Links}

Here are some useful links to software which can be used for analysis in structured populations:

EIGENSTRAT: http://genepath.med.harvard.edu/~reich/EIGENSTRAT.htm
GenABEL: http://mga.bionet.nsc.ru/~yurii/ABEL/
MACH: http://www.sph.umich.edu/csg/abecasis/mach/
PLINK: http://pngu.mgh.harvard.edu/~purcell/plink/

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\section*{Chapter 7}

\section*{GWA in presence of genetic stratification: practice}

Both ethnic admixture and presence of close relationships represents examples of confounding in association analysis. However, the methods to correct for stratification as resulting from mixture of subjects coming from different genetic populations, and methods to correct for family relations may be slightly different, and will be described separately in the next sections.

\subsection*{7.1 Analysis with ethnic admixture}

In previous section we detected genetic stratification by analysis of genomic kinship matrix and excluded genetic outliers from our further analysis. When there are only a few such outliers, exclusion them from analysis is a good option. However, in large studies cases and controls are usually selected across a number of locations and genetic populations, and stratification is expected by design. In such case, analysis of association should account for this stratification.

One of the ways to do that is to perform structured association analysis. In such analysis, effect and its variance are estimated within each strata separately, and then these estimates are pooled to generate global statistics. The strata can be known from design (e.g. place of birth or ethnicity of parents) or estimated from GWA data.

Let us do structured association analysis using the data1 data derived in previous section. First, we need to define the variable which will tell what population the study subjects belong to. In previous section, we stored the names of 'outlier' subjects in variable cl1:
```

> cl1

```
[1] "id2097" "id6954" "id2136" "id858"
We can use function \(\%\) in\% to find out what names of subjects are in cl1:
```

> pop <- as.numeric(data1@phdata\$id %in% cl1)
> pop

```

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[1] 0000000000000000000000001000000000000000000
[38] 00000000000000000000000000000000000000000
[75] 00000111200000000000000000000000000000000 [112] 00000000000000000000

Now, structured association may be done with qtscore function by specifying strata:
```

> data1.sa <- qtscore(dm2, data = data1, strata = pop)

```

We can compare results of analysis excluding outliers (black dots) and structured association analysis (green) by
```

> plot(data2.qt, cex = 0.5, pch = 19, ylim = c(1, 5))
> add.plot(data1.sa, col = c("lightgreen", "lightblue"), cex = 1.2)

```

The resulting plot is presented at figure 7.1 .
In this case, there is very little difference, because all people belonging to the smaller sub-population are cases.

Other way to adjust for genetic (sub)structure is to apply the method of Price et al., which make use of principal components of the genomic kinship matrix to adjust both phenotypes and genotypes for possible stratification. In GenABEL, such analysis is done using egscore function:
```

> tmp.gkin <- data1.gkin
> diag(tmp.gkin) <- hom(data1[, autosomal(data1)])\$Var
> data1.eg <- egscore(dm2, data = data1, kin = tmp.gkin)

```

Note that we have replaced the diagonal elements of the genomic kinship matrix, which are by default are \(0.5+\) th estimates of the inbreeding, by the variance returned by the hom function. The analysis plot may be added to the previous one by
```

> add.plot(data1.eg, col = c("red"), cex = 1.3)

```

The resulting plot is presented at figure 7.1
Again, the difference between three analysis methods is marginal because there are no highly differentiated SNPs in the data set, and one sub-population is presented by cases only. Still, the signals at chromosome one and three slightly improved, while these at two and X went down.

\section*{Exercise 13.}

Load and analyse the data set presented in file stratified.RData. GWA data presented in this file concern a study containing data from several popualtions. All these populations originate from the same base population some generations ago. Some of these populations mantained large size and some were small. There was little ( \(2.5 \%\) ) migration between populations.

Two traits (quat and bint) ara available for analysis. Investigate relations between phenotypes and covariates. Perform association analysis using structured association and egscore. Answer the questions
1. How many SNPs and IDs are presented in the data set?
2. How many SNPs and IDs pass the quality control?
3. Is there evidence for stratification coming from the distribution of GW test for HWE (what is lambda?)


Figure 7.1: Comparison of structured association analysis (green), method of Price et al. (red) and analysis excluding genetic outliers (solid black).
4. Is there evidence that the test statistics for trait quat is inflated (what is lambda?)
5. Is there evidence that the test statistics for trait bint is inflated (what is lambda?)
6. How many genetically distinct populations are present in the data set? How many people belong to each population?
7. Please make some inferences on the characteristics / history of these populations.
8. What is the strongest SNP associated with trait quat? What model (method and covariates used) gives best results? Is the finding GW-significant?
9. What is the strongest SNP associated with trait bint? What model (method and covariates used) gives best results? Is the finding GW-significant?

\subsection*{7.2 Analysis of family data}

In this section we will consider analysis of quantitative traits in a family-based cohort, where participants were not selected for the value of the trait under analysis. Such data may be generated in any study selecting participants based on kinship (e.g. collections of sibships, nuclear or extended families); also any study in a genetically isolated population is likely to end up with a large proportion of relatively closely related individuals, even if ascertainment was random with the respect to kinship.

In pedigree-based association analysis the pedigree works as a confounder exactly in the same manner as ethnic origin may work in a population-based study. Any genetic polymorphism is inherited through genealogy, and therefore
genotypes are more similar between close relatives. In the same manner, any other heritable trait will be also more similar between relatives, and therefore certain degree of association is expected between any genetic marker and any heritable trait in a family-based sample. If additive 1 d.f. test for association is considered, the effect of confounding by pedigree can be shown to inflate the resulting null distribution of presumably \(\chi_{1}^{2}\) test statistics by a certain constant \(\lambda\).

As you remember, this is exactly what happens when simple test for association is applied to a population-based data with ethnic admixture. In a population-based study with strong admixture (both in terms of the proportion and ethnic "distance"), some genomic regions may have been differentially selected in different populations. In such situation, use of genomic control does not prevent false-positive association between a trait and these regions, and other methods, such as EIGENSTRAT or Structured Association, are to be used.

For pedigree-based data coming from (relatively) genetically homogeneous population it can be shown that \(\lambda\) is a function of trait's heritability and pedigree structure, expressed as kinship matrix. Thus, genomic control is a simple and valid method to study association in genetically homogeneous families. However, this method reduces (or summarises if you prefer) all the abundant information about heritability and relationship into a single parameter \(\lambda\), therefore it is not the most powerful method.

In quantitative genetics, a mixed polygenic model of inheritance may be considered as "industrial standard" - this model has sound theoretical bases and is proven by time to describe well inheritance of complex quantitative traits. This model describes the vector of observed quantitative traits as
\[
\begin{equation*}
Y=\mu+G+e \tag{7.1}
\end{equation*}
\]
where \(\mu\) is the intercept, \(G\) is contribution from polygene, and \(e\) is random residual.

It is assumed that for each individual its "personal" random residual \(e_{i}\) is distributed as Normal with mean zero and variance \(\sigma_{e}^{2}\). As these residuals are independent between pedigree members, the joint distribution of residuals in the pedigree can be modelled using multivariate normal distribution with variance-covariance matrix proportional to the identity matrix \(I\) (this is a matrix with diagonal elements equal to 1 , and off-diagonal elements equal to zero): \(e \sim M V N\left(0, I \sigma_{e}^{2}\right)\).

The polygenic component \(G\) describes the contribution from multiple independently segregating genes all having a small additive effect onto the trait (infinitesimal model). For a person for whom parents are not known, it is assumed that \(G_{i}\) is distributed as Normal with mean zero and variance \(\sigma_{G}^{2}\). Assuming model of infinitely large number of genes, it can be shown that given polygenic values for parents, the distribution of polygene in offspring follows Normal distribution with mean \(\left(G_{m}+G_{f}\right) / 2\) and variance \(\sigma_{G}^{2} / 2\), where \(G_{m}\) is maternal and \(G_{f}\) is paternal polygenic values. From this, it can be shown that jointly the distribution of polygenic component in a pedigree can be described as multivariate normal with variance-covariance matrix proportional to the relationship matrix \(\Phi: G \sim M V N\left(0, \Phi \sigma_{G}^{2}\right)\).

Thus the log-likelihood for this model can be written as a function of three
parameters:
\[
\begin{align*}
L\left(\mu, \sigma_{G}^{2}, \sigma_{e}^{2}\right)= & -\frac{1}{2} \cdot \log _{e}\left|\left(\Phi \cdot \sigma_{G}^{2}+I \cdot \sigma_{e}^{2}\right)\right| \\
& +(Y-\mu)^{T} \cdot\left(\Phi \cdot \sigma_{G}^{2}+I \cdot \sigma_{e}^{2}\right)^{-1} \cdot(Y-\mu) \tag{7.2}
\end{align*}
\]
where \(\mu\) is intercept, \(\sigma_{G}^{2}\) is the proportion of variance explained by the polygenic component, and \(\sigma_{e}^{2}\) is the residual variance.

Covariates such as sex, age, or a genetic marker studied for association can be easily included into the model:
\[
Y=\mu+\sum_{j} \beta_{j} \cdot C_{j}+G+e
\]

Here, \(C_{j}\) is the vector containing \(j\)-th covariate and \(\operatorname{beta}_{j}\) is the coefficient of regression of \(Y\) onto that covariate.

This mixed morel leads to likleihood
\[
\begin{align*}
L\left(\mu, \sigma_{G}^{2}, \sigma_{e}^{2}, \beta_{1}, \beta_{2}, \ldots\right)= & -\frac{1}{2} \cdot \log _{e}\left|\left(\Phi \cdot \sigma_{G}^{2}+I \cdot \sigma_{e}^{2}\right)\right| \\
& +\left(Y-\left(\mu+\sum_{j} \beta_{j} \cdot C_{j}\right)\right)^{T} \cdot\left(\Phi \cdot \sigma_{G}^{2}+I \cdot \sigma_{e}^{2}\right)^{-1} \\
& \cdot\left(Y-\left(\mu+\sum_{j} \beta_{j} \cdot C_{j}\right)\right) \tag{7.3}
\end{align*}
\]

This general formulation can be easily adopted to test genetic association; for example, an effect of a SNP can be incorporated into regression model
\[
Y=\mu+\beta_{g} \cdot g+G+e
\]
where \(g\) is the vector containing genotypic values. In this mode, you can specify a variety of 1 d.f. models by different coding of the vector \(g\). For example, if you consider an "AG" polymorphism and want to estimate and test additive effect of the allele "G", you should code "AA" as 0 (zero), "AG" as 1 and "GG" as 2. Under this coding, the \(\beta_{g}\) will estimate additive contribution from the " G " allele. If you are willing to consider dominant model for \(G\), you should code "AA" and "AG" as 0 and "GG" as 1. Recessive and over-dominant models can be specified in a similar manner. If, however, you want to estimate general 2 d.f. model, the specification should be different:
\[
Y=\mu+\beta_{a} \cdot g+\beta_{d} \cdot I_{g=2}+G+e
\]
where \(g\) is coded as 0,1 or 2 , exactly the same as in the additive model, and \(I_{g=2}\) is the binary indicator which takes value of one when \(g\) is equal to 2 and zero otherwise. In this model, \(\beta_{a}\) will estimate the additive and \(\beta_{d}\) - the dominance effect. There may be other, alternative coding(s) allowing for essentially the same model, for example
\[
Y=\mu+\beta_{1} \cdot I_{g=1}+\beta_{2} \cdot I_{g=2}+G+e
\]
would estimate trait's deviation in these with \(g=1\left(\beta_{1}\right)\) and these with \(g=2\) \(\left(\beta_{2}\right)\) from the reference \((g=0)\).

The classical way to estimate mixed polygenic model and test for significance is Maximum Likelihood (ML) or Restricted ML (REML) using equation (7.3). However, when large pedigrees are analysed, ML/REML solution may take prohibitively long time, i.e. from minutes to hours for single SNP analysis, making study of hundreds of thousand of SNPs impossible. Therefore fast approximate tests were developed for the purposes of GWA association analysis in samples of relatives.

Here we will cover two of fast approximations available, FAmily-based Score Test for Association (FASTA, Chen \& Abecasis, 2007) and Genome-wide Rapid Analysis using Mixed Models And Score test (GRAMMAS, Amin et al, 2007). Both tests are based on the classical polygenic mixed model and are performed in two steps.

First, polygenic model as specified by equation (7.1) and likelihood (7.2) is estimated using available data.

Secondly, the maximum likelihood estimates (MLEs) of the intercept, \(\hat{\mu}\), proportion of variance explained by the polygenic component, \(\hat{\sigma}_{G}^{2}\), and residual variance, \(\hat{\sigma}_{e}^{2}\), are used to compute the FASTA test statistics
\[
T_{F}^{2}=\frac{\left((g-E[g])^{T} \cdot\left(\Phi \cdot \hat{\sigma}_{G}^{2}+I \cdot \hat{\sigma}_{e}^{2}\right)^{-1} \cdot(Y-\hat{\mu})\right)^{2}}{(g-E[g])^{T} \cdot\left(\Phi \cdot \hat{\sigma}_{G}^{2}+I \cdot \hat{\sigma}_{e}^{2}\right)^{-1} \cdot(g-E[g])}
\]

It can be shown that \(T_{F}^{2}\) follows \(\chi_{1}^{2}\) when pedigree structure is \(100 \%\) complete and \(100 \%\) correct. As this is never actually the case, application of GC to correct for residual inflation is recommended.

FASTA test results in unbiased estimates of the SNP effect and correct \(P-\) values. Please keep in mind that this is correct - as for any score test - only when alternative is reasonably close to the null, i.e. when the SNP explains small proportion of trait's variance. Disadvantages of this test are that is can be relatively slow when thousands of study subjects are analysed, and that permutation procedures can not be applied to estimate genome-wide significance, because the data structure is not exchangeable.

Other test, GRAMMAS, also exploits MLEs from the polygenic model 7.1. However, these are used to first compute the vector of environmental residuals \(\hat{e}\), using standard equation
\[
\hat{e}=\hat{\sigma}_{e}^{2} \cdot\left(\Phi \cdot \hat{\sigma}_{G}^{2}+I \cdot \hat{\sigma}_{e}^{2}\right)^{-1} \cdot(Y-\hat{\mu})
\]

These residuals, in turn, are used to run simple score test:
\[
T_{G}^{2}=\frac{\left.\left((g-E[g])^{T} \cdot \hat{e}\right)\right)^{2}}{(g-E[g])^{T} \cdot(g-E[g])}
\]

This test is conservative, but GC can be used to correct for the deflation of the test statistics.

The fact that environmental residuals \(\hat{e}\) are not dependent on pedigree structure leads to a nice property of the GRAMMAS test: the data structure becomes exchangeable and permutations may be used to estimate genome-wide significance. When used in combination with GC, \(P\)-values derived from

GRAMMAS test are correct; however, there is a downward bias in estimates of SNP effects.

When using FASTA or GRAMMAS test, it is recommended to estimate genomic kinship matrix from available genome-wide data, and use it in analysis instead of pedigree kinship. This solution firstly does not rely on the completeness and quality of pedigree; secondly, genomic kinship is more likely to give a better estimate of a true covariance between individual genomes, while pedigree kinship provides one with expectation. Therefore use of genomic kinship is expected to lead to better estimates of polygenic model, and thus better power to detect association in GWA analysis. This being said, we generally advocate use of genomic, and not pedigree kinship. Of cause, you can only implement this solution when you have GWA data; in a candidate gene study you will have to rely on the pedigree structure to estimate kinship matrix.

\subsection*{7.3 Example GWA analysis using family-based data}

In this section, we will explore small data set (150 people, 5827 SNPs). Let us load and explore it:
```

> load("RData/erfsmall.RData")
> ls()
[1] "erfs" "pkins"
> class(erfs)
[1] "gwaa.data"
attr(,"package")
[1] "GenABEL"
> class(pkins)
[1] "matrix"

```

You can see that there a two objects, erfs and pkins, presented in the data. The class of the first object is standard GenABEL 's gwaa.data-class; this is the object containing GWA data. The other object contains kinship matrix, as estimated from pedigree data.

You can check the number of people and SNPs in the data set with
```

> erfs@gtdata@nids

```
[1] 150
> erfs@gtdata@nsnps
[1] 5827
As usual, it is advisable to check the distribution of SNPs by chromosome:
```

> table(erfs@gtdata@chromosome)

```

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\begin{tabular}{rrrrrrrrrrrrrrrrrrrr}
1 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 2 & 20 & 21 & 22 & 23 & 3 & 4 & 5 & 6 \\
484 & 251 & 224 & 285 & 188 & 210 & 206 & 189 & 159 & 170 & 149 & 481 & 130 & 100 & 132 & 26 & 403 & 300 & 320 & 397 \\
7 & 8 & 9 & X & & & & & & & & & & & & & & & & \\
286 & 253 & 206 & 278 & & & & & & & & & & & & & & & & \\
\end{tabular}
(here, 23 stays for pseudo-autosomal region of the X chromosome); you can see that markers are evenly spread over the chromosomes.

Summary marker statistics can be generated by
```

> descriptives.marker(erfs@gtdata)
\$`Minor allele frequency distribution`
X<=0.01 0.01<X<=0.05 0.05<X<=0.1 0.1<X<=0.2 X>0.2
No 17.000 26.000 75.000 437.000 5272.000
llllll
\$`Cumulative distr. of number of SNPs out of HWE, at different alpha`
X<=1e-04 X<=0.001 X<=0.01 X<=0.05 all X
No 2 14.000 83.000 319.000 5827
Prop 0 0.002 0.014 0.055 1
\$`Distribution of porportion of successful genotypes (per person)`
X<=0.9 0.9<X<=0.95 0.95<X<=0.98 0.98<X<=0.99 X>0.99
No 0 1.000 2.000 12.00 135.0
llllll
\$`Distribution of porportion of successful genotypes (per SNP)`
X<=0.9 0.9<X<=0.95 0.95<X<=0.98 0.98<X<=0.99 X>0.99
No 77.000 33.000 214.000 208.000 5295.000
Prop 0.013 0.006 0.037 0.0.036 0.909
\$`Mean heterozygosity for a SNP`
[1] 0.4402752
\$`Standard deviation of the mean heterozygosity for a SNP`
[1] 0.08287253
\$`Mean heterozygosity for a person`
[1] 0.4354001
\$`Standard deviation of mean heterozygosity for a person`
[1] 0.01305448

```

You can see that the quality of genotypic data is quite reasonable: call rate is generally high, both per-person and per SNP, and there is little deviation from Hardy-Weinberg equilibrium.

Let us explore pedigree kinship matrix. First, let us just look how this matrix looks like by displaying few elements from the upper-left corner:
```

> pkins[1:5, 1:5]

```
```

id1 id2 id3 id4 id5
id1 5.00000e-01 8.56146e-05 1.01984e-04 2.33397e-04 8.56146e-05
id2 8.56146e-05 5.00000e-01 3.96513e-03 2.56896e-05 2.51269e-01
id3 1.01984e-04 3.96513e-03 5.00000e-01 1.21593e-05 3.96513e-03
id4 2.33397e-04 2.56896e-05 1.21593e-05 5.00000e-01 2.56896e-05
id5 8.56146e-05 2.51269e-01 3.96513e-03 2.56896e-05 5.00000e-01

```

By definition, pedigree kinship should take values between 0 and 0.5 (plus some small amount from inbreeding); kinship between (non-inbred) sibs or an offspring and the parent is \(1 / 4\). You can see that in the upper-left corner there is one inbred sib-pair (or parent-offspring pair; "id2" and "id5"). You can also see that this matrix is symmetric around the diagonal.

Let us summarise the distribution of kinship coefficients; in doing this we want to generate the summary for every off-diagonal element only once. Function lower.tri can be used to get the "lower triangle" sub-matrix elements:
```

> summary(pkins[lower.tri(pkins)])

```

Min. 1st Qu. Median Mean 3rd Qu. Max. 0.00000000 .00049990 .00285000 .00628100 .00537800 .2633000

As you can see, average relationship corresponds to that expected between second cousins \((1 / 64=0.015625)\) and third cousins \((1 / 256=0.00390625)\).

We can also draw a histogram of the distribution of the kinship coefficients (shown at figure 7.2 A):
```

> hist(pkins[lower.tri(pkins)])

```
and see that most elations are indeed remote.
Let us estimate genomic kinship matrix using autosomal data with the command ibs, and look up the elements in the upper-left corner:
```

> gkins <- ibs(erfs[, autosomal(erfs)], weight = "freq")
> gkins[1:5, 1:5]

```
id1 id2 id3 id4 id5
id1 \(0.526167668 \quad 5.439000 e+03 \quad 5.446000 e+03 \quad 5441.000000005440 .0000000\)
id2 -0.012569446 4.863918e-01 \(5.524000 e+035524.000000005521 .0000000\)
id3 \(0.001516184-8.551323 \mathrm{e}-03 \quad 5.088129 \mathrm{e}-01 \quad 5529.000000005528 .0000000\)
id4 \(0.010459422-1.471218 \mathrm{e}-02-3.467894 \mathrm{e}-03 \quad 0.514796475523 .0000000\)
id5 -0.007957596 \(2.561484 e-01-8.925127 e-03 \quad-0.02205665 \quad 0.4886211\)
Here, the estimated kinship is shown below the diagonal, and the number of informative SNP pairs used for estimation is shown above the diagonal.

You can see that "genomic kinship" coefficients may take values lower than zero, which is consequence of the fact that in effect "genomic kinship" is simply covariance between the vectors of individual genotypes. This quantity, though it provides an unbiased estimate of kinship, can be lower than zero.
```

> summary(gkins[lower.tri(gkins)])
Min. 1st Qu. Median Mean 3rd Qu. Max.
-0.038980 -0.011930-0.005826 -0.003362 0.000481 0.268000

```


Figure 7.2: Distribution of the pedigree (upper histogram) and genomic (lower histogram) kinship coefficients for erfs data set.
here, the average is quite close to that obtained with pedigree kinship.
We can also draw a histogram of the distribution of "genomic kinship" coefficients (shown at figure 7.2 B):
> hist(gkins[lower.tri(gkins)])
and can easily graphically present relations between genomic and pedigree kinship with
```

> plot(pkins[lower.tri(pkins)], gkins[lower.tri(gkins)])

```
(shown at figure 7.3), and estimate correlation between the two with
```

> cor(pkins[lower.tri(pkins)], gkins[lower.tri(gkins)])

```

\section*{[1] 0.91615}

From the graph, you can clearly see that, though there is a very strong correlation between genomic and pedigree kinships, these are not identical.

In real data, you may find that there are some points where pedigree data clearly suggest relation different from that suggested by genomic data. Which one to believe? Generally, pedigrees are more prone to errors than genotypic data. In the data containing close relatives it is better to rely on "genomic kinship".

Let us first analyse the data using plain GC method:


Figure 7.3: Scatter-plot relating pedigree and genomic kinships for erfs data set.
```

> qts <- qtscore(qtbas, data = erfs)

```

You can check the estimate of the inflation factor \(\lambda\) with
```

> qts$lam$est

```
[1] 1.137247
This is relatively high value, suggesting presence of close relatives in data and high heritability of the trait.

The top 10 hits from GWA analysis can be displayed with
```

> descriptives.scan(qts, sort = "Pc1df")

```
\begin{tabular}{lrrrrrr} 
& Chromosome & Position & N & effB & P1df & Pc1df \\
rs1781670 & 13 & 2694164735 & 150 & 0.4554848 & 0.0001804816 & 0.0004453466 \\
rs1054889 & 2 & 324154337 & 150 & 0.4760959 & 0.0002711808 & 0.0006387800 \\
rs1861659 & 16 & 3171591122 & 150 & 0.4844231 & 0.0002811280 & 0.0006594988 \\
rs1860991 & 5 & 1213061568 & 150 & 0.4616985 & 0.0002834991 & 0.0006644252 \\
rs1043883 & 8 & 1745120471 & 128 & 0.4338657 & 0.0005729702 & 0.0012396586 \\
rs1872087 & 17 & 3329938025 & 150 & -0.4111272 & 0.0006091602 & 0.0013088424 \\
rs1982904 & 4 & 961162979 & 150 & 0.5369605 & 0.0006904958 & 0.0014626936 \\
rs2141693 & 12 & 2590653716 & 150 & 1.2178255 & 0.0007070577 & 0.0014937647 \\
rs7141672 & 14 & 2851979738 & 150 & 0.4595401 & 0.0007473279 & 0.0015689781
\end{tabular}

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\begin{tabular}{|c|c|c|c|c|}
\hline rs1075456 & 15 & 3036078968 & \(150-0.3943230\) & 0.00080947640 .0016841742 \\
\hline & effab & effBB & P2df & \\
\hline rs1781670 & 0.55710736 & 0.9015131 & 0.0007574863 & \\
\hline rs1054889 & 0.32660599 & 1.0001217 & 0.0008416365 & \\
\hline rs1861659 & 0.30184684 & 1.0290361 & 0.0006959544 & \\
\hline rs1860991 & 0.49412227 & 0.9080701 & 0.0013452354 & \\
\hline rs1043883 & 0.23834763 & 0.9427557 & 0.0015609770 & \\
\hline rs1872087 & -0.50149169 & -0.8092418 & 0.0024403173 & \\
\hline rs1982904 & 0.60470761 & 0.8678924 & 0.0026214066 & \\
\hline rs2141693 & 1.21782551 & NA & 0.0007070577 & \\
\hline rs7141672 & 0.37923110 & 1.0172357 & 0.0028443093 & \\
\hline rs1075456 & -0.01486038 & -0.8540990 & 0.0002978615 & \\
\hline
\end{tabular}
here, nominal \(P\) - values after genomic control are given in column named "Pc1df".

We can estimate genome-wide empirical significance by using the same function with times argument, which tells the number of permutations:
```

> qts.e <- qtscore(qtbas, data = erfs, times = 200, quiet = TRUE)

```
> descriptives.scan(qts.e, sort = "Pc1df")
\begin{tabular}{lrrrrrrr} 
& Chromosome & Position & N & effB & P1df & Pc1df & effAB \\
rs1781670 & 13 & 2694164735 & 150 & 0.45548481 & 0.555 & 0.825 & 0.5571074 \\
rs1054889 & 2 & 324154337 & 150 & 0.47609594 & 0.670 & 0.890 & 0.3266060 \\
rs1861659 & 16 & 3171591122 & 150 & 0.48442305 & 0.680 & 0.895 & 0.3018468 \\
rs1860991 & 5 & 1213061568 & 150 & 0.46169852 & 0.680 & 0.900 & 0.4941223 \\
rs1043883 & 8 & 1745120471 & 128 & 0.43386568 & 0.880 & 0.990 & 0.2383476 \\
rs1872087 & 17 & 3329938025 & 150 & -0.41112719 & 0.890 & 0.995 & -0.5014917 \\
rs2017143 & 1 & 2252811 & 150 & 0.06610235 & 1.000 & 1.000 & 0.3084208 \\
rs2840531 & 1 & 2318570 & 150 & -0.31150314 & 1.000 & 1.000 & -0.3273450 \\
rs2477703 & 1 & 2458154 & 150 & -0.16765976 & 1.000 & 1.000 & -0.2043520 \\
rs734999 & 1 & 2545378 & 150 & -0.04010750 & 1.000 & 1.000 & 0.2438654
\end{tabular}
            effBB P2df
rs1781670 0.901513130 .970
rs1054889 1.000121700 .990
rs1861659 1.029036070 .965
rs1860991 0.908070111 .000
rs1043883 0.942755731 .000
rs1872087-0.80924177 1.000
rs2017143 0.053428491 .000
rs2840531-0.47957726 1.000
rs2477703-0.29647369 1.000
rs734999 -0.09548236 1.000
(argument "quiet" supress warning messages; this is used for the purposes of this tutorial, normally you do not need to specify this option)

As you can see, in this analysis nothing comes even close to genome-wide significance, as indicated by genome-wide corrected \(P\)-values (column "Pc1df") all \(\gg 0.05\).

Let us try to estimate polygenic model with
```

> h2 <- polygenic(qtbas, kin = gkins, data = erfs)

```

The results of estimation are contained in "h2an" element of the resulting analysis object:
```

> h2\$h2an
\$minimum
[1] 140.0211
\$estimate
[1] 0.03828502 0.76623275 1.21485871
\$gradient
[1] -3.8304294 0.2006816 -3.9893741
\$code
[1] 2
\$iterations
[1] 4

```

In the "estimate" list, the MLEs shown correspond to intercept \(\hat{\mu}\), heritability \(\hat{h}^{2}=\hat{\sigma}_{G}^{2} /\left(\hat{\sigma}_{G}^{2}+\hat{\sigma}_{e}^{2}\right)\), and total variance \(\hat{\sigma}_{T}^{2}=\hat{\sigma}_{G}^{2}+\hat{\sigma}_{e}^{2}\). You can see that heritability of the trait is indeed high - almost \(80 \%\).

Under these conditions (hight heritability, presence of close relatives) we may expect that FASTA and GRAMMAS analysis exploiting heritability model and relationship matrix in exact manner may have better power compared to simple GC.

Let us run FASTA test using estimated polygenic model, as specified by h2 object:
```

> mms <- mmscore(h2, data = erfs)

```

There is little residual inflation left when we use "genomic kinship" matrix:
```

> mms$lam$est

```
[1] 1.028614
And the significance of "top" hit becomes an order of magnitude better compared to plain GC:
```

> descriptives.scan(mms, sort = "Pc1df")

```
\begin{tabular}{lrrrrrrr} 
& Chromosome & Position & N & effB & P1df & Pc1df & effAB \\
rs1075456 & 15 & 3036078968 & 150 & -0.5367991 & 0.0000117915 & 0.0000156003 & NA \\
rs1054889 & 2 & 324154337 & 150 & 0.4977818 & 0.0001995063 & 0.0002448961 & NA \\
rs1781670 & 13 & 2694164735 & 150 & 0.4722558 & 0.0002277562 & 0.0002786016 & NA \\
rs1264007 & X & 4256199578 & 150 & -0.3768665 & 0.0006969348 & 0.0008279231 & NA \\
rs774033 & 12 & 2531521679 & 150 & -0.4282825 & 0.0009133647 & 0.0010774162 & NA \\
rs7141672 & 14 & 2851979738 & 150 & 0.4605324 & 0.0010423648 & 0.0012253707 & NA \\
rs537848 & 15 & 3020539910 & 150 & 0.4149924 & 0.0011712404 & 0.0013727112 & NA
\end{tabular}

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\begin{tabular}{|c|c|c|c|c|c|}
\hline rs760109 & X & 4353106270150 & 0.36694140 .0013345180 & 0.0015587896 & NA \\
\hline rs1860991 & 5 & 1213061568150 & 0.41182290 .0013704970 & 0.0015997116 & NA \\
\hline rs1370139 & 6 & 1381958118149 & 0.58986930 .0016150035 & 0.0018771154 & NA \\
\hline & effBB P2df & & & & \\
\hline rs1075456 & NA 9.99 & & & & \\
\hline rs1054889 & NA 9.99 & & & & \\
\hline rs1781670 & NA 9.99 & & & & \\
\hline rs1264007 & NA 9.99 & & & & \\
\hline rs774033 & NA 9.99 & & & & \\
\hline rs7141672 & NA 9.99 & & & & \\
\hline rs537848 & NA 9.99 & & & & \\
\hline rs760109 & NA 9.99 & & & & \\
\hline rs1860991 & NA 9.99 & & & & \\
\hline rs1370139 & NA 9.99 & & & & \\
\hline
\end{tabular}

If you compare these results to that obtained with simple GC, you can also see that the ranks of top hits have changed quite a bit; unbiased estimated of genetic effects were obtained.

However, we can not estimate genome-wide significance with FASTA, because the data structure is not exchangeble.

Using GRAMMAS method, you can estimate nominal \(P-\) values by
```

> grs <- qtscore(h2$pgres, data = erfs, clam = FALSE)
> grs$lam\$est

```

\section*{[1] 0.8018405}

In the above analysis, note that the estimated "inflation" factor \(\lambda\) is less than one, i.e. now it is the GRAMMAS deflation factor. In order to obtain non-concervative test statistics, we had to say to qtscore that deflation is OK (parameter clam=FALSE).

We can see "top" nominal corrected \(P\) - values with
```

> descriptives.scan(grs, sort = "Pc1df")

```
\begin{tabular}{lrrrrrr} 
& Chromosome & Position & N & effB & P1df & Pc1df \\
rs1075456 & 15 & 3036078968 & 150 & -0.11663795 & 0.0001330355 & 0.0000198256 \\
rs1054889 & 2 & 324154337 & 150 & 0.11181723 & 0.0009735394 & 0.0002304027 \\
rs1781670 & 13 & 2694164735 & 150 & 0.10016312 & 0.0014943414 & 0.0003903064 \\
rs1264007 & \(X\) & 4256199578 & 150 & -0.08473388 & 0.0023587797 & 0.0006839906 \\
rs774033 & 12 & 2531521679 & 150 & -0.10130246 & 0.0025529578 & 0.0007537927 \\
rs7141672 & 14 & 2851979738 & 150 & 0.10205691 & 0.0038821895 & 0.0012610284 \\
rs1860991 & 5 & 1213061568 & 150 & 0.09471401 & 0.0040832449 & 0.0013416220 \\
rs537848 & 15 & 3020539910 & 150 & 0.09271600 & 0.0041030542 & 0.0013496117 \\
rs760109 & X & 4353106270 & 150 & 0.08107110 & 0.0045301478 & 0.0015239225 \\
rs2639197 & 15 & 3068765468 & 150 & -0.0992481 & 0.0051383932 & 0.0017785299 \\
& effAB & effBB & P2df & & \\
rs1075456 & -0.05270565 & -0.2443035 & 0.0002344231 & & \\
rs1054889 & 0.04844500 & 0.2439530 & 0.0013041771 & & \\
rs1781670 & 0.08836984 & 0.2014237 & 0.0062352965 & & \\
rs1264007 & -0.10721061 & -0.1650157 & 0.0097603773 & & &
\end{tabular}
```

rs774033 -0.10291990 -0.2025575 0.0105487887
rs7141672 0.09313608 0.2150170 0.0149681884
rs1860991 0.11795088 0.1784438 0.0135843361
rs537848 0.09617266 0.1851661 0.0162155918
rs760109 0.06708052 0.1627115 0.0177638794
rs2639197-0.10031663-0.1971147 0.0199290615

```

By comparing this output to that from FASTA test, you can see that \(P\) values are quite close, but the effects are underestimated with GRAMMAS, as expected.

However, the streangths of GRAMMAS test is not only its speed, but also possiblity to estimate genome-wide significance. This can be done by
```

> grs.e <- qtscore(h2\$pgres, data = erfs, times = 200, clam = FALSE,

+ quiet = TRUE)

```
> descriptives.scan(grs.e, sort = "Pc1df")
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline & Chromosome & Position & N & effB & P1df & Pc1df & effAB \\
\hline rs1075456 & 15 & 3036078968 & 150 & -0.11663795 & 0.410 & 0.075 & -0.05270565 \\
\hline rs1054889 & 2 & 324154337 & 150 & 0.11181723 & 0.965 & 0.575 & 0.04844500 \\
\hline rs1781670 & 13 & 2694164735 & 150 & 0.10016312 & 0.995 & 0.780 & 0.08836984 \\
\hline rs1264007 & X & 4256199578 & 150 & -0.08473388 & 1.000 & 0.920 & -0.10721061 \\
\hline rs774033 & 12 & 2531521679 & 150 & -0.10130246 & 1.000 & 0.935 & -0.10291990 \\
\hline rs1860991 & 5 & 1213061568 & 150 & 0.09471401 & 1.000 & 0.985 & 0.11795088 \\
\hline rs7141672 & 14 & 2851979738 & 150 & 0.10205691 & 1.000 & 0.985 & 0.09313608 \\
\hline rs537848 & 15 & 3020539910 & 150 & 0.09271600 & 1.000 & 0.985 & 0.09617266 \\
\hline rs760109 & X & 4353106270 & 150 & 0.08107110 & 1.000 & 0.995 & 0.06708052 \\
\hline rs2017143 & 1 & 2252811 & 150 & 0.02573273 & 1.000 & 1.000 & 0.05910715 \\
\hline & effBB & P2df & & & & & \\
\hline rs1075456 & -0.24430347 & 0.515 & & & & & \\
\hline rs1054889 & 0.24395303 & 0.990 & & & & & \\
\hline rs1781670 & 0.20142366 & 1.000 & & & & & \\
\hline rs1264007 & -0.16501574 & 1.000 & & & & & \\
\hline rs774033 & -0.20255749 & 1.000 & & & & & \\
\hline rs1860991 & 0.17844379 & 1.000 & & & & & \\
\hline rs7141672 & 0.21501704 & 1.000 & & & & & \\
\hline rs537848 & 0.18516610 & 1.000 & & & & & \\
\hline rs760109 & 0.16271149 & 1.000 & & & & & \\
\hline rs2017143 & 0.04061566 & 1.000 & & & & & \\
\hline
\end{tabular}

As you can see, now the "top" hit starts approaching genome-wide significance (genome-wide \(P-\) value \(\sim 10 \%\) ), showing the power of kinship-based methods under high heritability model.

Finally, let us plot - \(\log _{10}\) nominal \(P\)-values from different methods across the genome. Let black dots correspond to GC, green to GRAMMAS and red to FASTA (figure 7.4):
```

> plot(mms, df = "Pc1df")
> add.plot(grs, df = "Pc1df", col = c("lightgreen", "lightblue"),

```
```

mmscore(h2, erfs)

```


Figure 7.4: Comparison of FASTA (red), GRAMMAS (green), and plain GC (black).
```

+ cex = 1.2)
> add.plot(qts)

```

You can see that there is a great degree of correlation between the FASTA and GRAMMAS \(P\) - values, while plain GC really stands apart.

\subsection*{7.4 Exercise: analysis of family data}

Exercise 30 Repeat heritability estimation, FASTA and GRAMMAS analysis of previous section using pediree kinship (pkins object). Discuss the results.

In the next section, you will explore a small ( 695 people) subset of people from ERF, a family-based study with participants coming from a genetically isolated population and sampled based on kinship (all living descendants of 22 couples living in the area in mid-XIX \({ }^{t h}\) century). The study participants were genotyped using Illumina 6K "linkage" array. QC was already performed. Your trait of interest is "qtbas".

Explore the data set and answer the questions:
Exercise 31 Describe the trait "qt". Can you detect significant outliers at visual inspection? Is trait distributed normally? What are significant covariates?

Exercise 32 Explore relations between genomic and pedigree kinship (these are provided in data as gkin and pkin data objects, respectively). What are your conclusions? Which matrix would you use later on?

Exercise 33 What is the heritability of the trait (take care: polygenic analysis may rung for a long while)? Based on heritability analysis, how would you rank different methods of GWA analysis for this trait (and why)?

Exercise 34 Do GWA analysis using simple score test with genomic control. Estimate genome-wide significance. What are your conclusions?

Run GWA analysis using the "best" method and model as you have decided in previous exercises. Estimate genome-wide significance. What are your conclusions? Did they change compared to simple analysis?

Exercise 35 Repeat the last "best" analysis using pedigree kinship. How your results change?

Exercise 36 If you have any time left - repeat analysis using "qt" trait. This one is much more fun, but also more laborous to analyse.

\section*{Chapter 8}

\section*{Exploring and using public databases}

More functions coming soon.

\section*{Chapter 9}

\section*{Genetic data imputations}

With very dense SNP panels available nowadays, linkage disequilibrium between neighbouring SNP is high. This allows us, based on the context, predict genotypes with high degree of accuracy. The genetic data imputations refer to such prediction of genotypes of missing SNPs,

The uses of genetic data imputations are basically two-fold. The first situation concerns imputations of partly missing genotypes of SNPs, typed, in a study, based on the context provided by SNPs typed in the same study. This allows decreasing the proportion of missing SNPs, potentially improving the power of the study. The second situation concerns prediction of genotypes at SNPs, which were not typed in the study, based on the context provided by the SNPs typed and using the data coming from another study with more dense genotyping, which may be potentially done using other population. Though imputations may be done using the same method under the two scenarios, and difference between them is quantitative rather than qualitative, for practical reasons it is useful to distinguish these two situations.

GenABEL is well connected with MACH imputations software, developed by Y. Li and G. Abecasis. Please read abou MACH at http://www.sph.umich.edu/csg/abecasis/MACH/.

\subsection*{9.1 Imputing partly missing genotypes}

Assume you have gwaa.data object "data" and you want to impute missing genotypes for chromosome 1. For this, you will first need to export chromosome one data in MACH format. The MACH data format bears remarkable similarity to the format used by MERLIN, an excellent linkage analysis program. This is not surprising: MERLIN was developed by G. Abecasis. Therefore we can export the data for MACH using our function export.merlin:
```

> export.merlin(data[,data@gtdata@chromosome=="1"],pedf="chr1.ped",
+
dataf="chr1.dat",mapf="chr1.map")

```
where the first argument supplies the data to be exported and others specify the names for the output files containing MACH genetic data (pedigree-file), and descriptives (data-file). It will also save map information in file 'chr1.map'; this information is not necessarily used by MACH, but is required by MERLIN, and also required by GenABEL .

Given the data are exported, we can use MACH for imputations of missing genotypes. Read MACH documentation for details. Basically, under UNIX, you need to run command
```

bash> mach1 -p chr1.ped -d chr1.dat --states 200 --rounds 20
--mle --mldetails --prefix chr1.out

```

This should produce files, of which we will use chr1.out.mlgeno, containing the imputed genotypes and chr1.out.mlinfo, containing information about imputations process, including quality score.

Now we need to read the imputed data back into R environment. First, we need convert MACH output to GenABEL format:
```

> convert.snp.mach(pedf="chr1.out.mlgeno",mapf="chr1.map",

+ info="chr1.out.mlinfo",outf="chr1.raw",quality=0.9)

```

This will convert MACH output to file 'chr1.raw', containing genotypic data in GenABEL format. All SNPs with quality \(<0.9\) will be dropped.

Conversion of data from MACH to GenABEL format may be a lengthy procedure. You can improve the speed much by replacing all "/" to space (" ") in 'chr1.out.mlgeno'.

As soon as conversion is done, you can import the data by
```

> chr1.imp <- load.gwaa.data(phe="pheno.dat",gen="chr1.raw")

```

This assumes that your penotypes are in file "pheno.dat", probably easiest way to generate it is by
```

> save.gwaa.data(data,phen="pheno.dat",geno="tmp.raw")

```

Later, you can even replace your original data with imputed:
```

> data@gtdata@gtps[,data@gtdata@chromosome=="1"] <- chr1.imp@gtdata@gtps

```

\subsection*{9.2 Inferences from other data sets}

If you imported your data to GenABEL before version 1.2-6, you will need to re-load your data to GenABEL, to provide the information about actual coding and strand (easily available for Affymetrix). Please check documentation for convert.snp.text, convert.snp.illumina, convert.snp.ped. For the latter two take care that strand="file".

Assume that your data are in gwaa.data object data; you want to impute SNPs in the region between SNP 100 and 500.

Go to HapMap web-site and download phased haplotype data for the region between SNP 100 and 500 . We have a script which generates MACH input files from HapMap dump-files (dump2mach.pl, available upon request). Note that in HapMap phased haplotype data, SNP coding is given for " + " strand.

Assume that haplotype and SNP file names are reg1.haplo, reg1.snps (read MACH documentation for details). Check that all of your SNPs (from 100 to 500) are in HapMap data; else generate the vector of indexes/names of the ones which are in HapMap (only these will pass through MACH). Assuming all 500 are in HapMap, export your regional data in MACH format by:
```

> export.merlin(data[,c(100:500)],pedf="reg1.ped",

+ dataf="reg1.dat",mapf="reg1.map",strand="+")

```

To do imputations, in bash type
```

bash> mach1 -p reg1.ped -d reg1.dat -h reg1.haplo -s reg1.snps
--greedy --rounds 20 --mle --mldetails --prefix reg1.out

```

This should produce files, of which we will use reg1.out.mlgeno, containing the imputed genotypes, and reg1.out.mlinfo, containing information about imputations process, including quality score.

Back in R, convert the data to GenABEL format by:
```

> convert.snp.mach(pedf="reg1.out.mlgeno",mapf="reg1.large.map",

+ info="reg1.out.mlinfo",outf="reg1.raw",quality=0.9)

```

Here, reg1.large.map provides map for all HapMap SNPs in the region (this one is easily made from the dump-file, or with dump2mach.pl) The above command will convert MACH output to GenABEL format, dropping all SNPs with quality \(<0.9\).

Conversion of data from MACH to GenABEL format may be a lengthy procedure. You can improve the speed much by replacing all "/" to space (" ") in 'chr1.out.mlgeno'.

Now, you can import the data by
```

> reg1.imp <- load.gwaa.data(phe="pheno.dat",gen="reg1.raw")

```

\section*{Chapter 10}

\section*{Imperfect knowledge about genotypes}

This section is copy-pasted from ProbABEL manual - will be re-written.
Many statistical and experimental techniques, such as imputations and highthroughput sequencing, generate the data, which are informative for genomewide association analysis, and are probabilistic in the nature.

When we work with directly genotyped markers using such techniques as SNP or microsatellite typing, we would normally know the genotype of a particular person at a particular locus with very high degree of confidence, and, in case of biallelic marker, can state whether genotype is \(A A, A B\) or \(B B\).

On the contract, when dealing with imputed or high-throughput sequencing data, for many of genomic loci we are quite uncertainty about genotypic status of the person; instead of known genotypes we deal rather with probability distribution; that is based on observed information, we have estimates that true underlying genotype is either \(A A, A B\) or \(B B\); the degree of confidence about the real status is measured with probability distribution \(\{P(A A), P(A B), P(B B)\}\).

Several techniques may be applied to analyse such data. The most simplistic approach would be to pick up the genotype with highest probability, i.e. \(\max _{g}[P(g=A A), P(g=A B), P(g=B B)]\) and then analyse the data as if directly typed markers were used. The disadvantage of this approach is that it does not take into account the probability distribution - i.e. the uncertainty about the true genotypic status. Such analysis is statistically wrong: the estimates of association parameters (regression coefficients, odds or hazard ratios, etc.) are biased, and the bias becomes more pronounced with greater probability distribution uncertainty (entropy).

One of the solution which generates unbiased estimates of association parameters and takes probability distribution into account is achieved by performing association analysis by means of regression of the outcome of interest onto estimated genotypic probabilities.

\subsection*{10.1 Regression analysis}

Standard linear theory is used to estimate betas and their standard errors. We assume linear model with expectation
\[
\begin{equation*}
E[\mathbf{Y}]=\mathbf{X} \beta \tag{10.1}
\end{equation*}
\]
and variance-covaraince matrix
\[
\mathbf{V}=\sigma^{2} \mathbf{I}
\]
where \(\mathbf{Y}\) is the vector of phenotypes of interest, \(\mathbf{X}\) is design matrix, \(\beta\) is the vector of regression parameters, \(\sigma^{2}\) is variance and \(\mathbf{I}\) is identity matrix.

The maximum likelihood estimates (MLEs) for the regression parameters is given by
\[
\begin{equation*}
\hat{\beta}=\left(\mathbf{X}^{T} \mathbf{X}\right)^{-1} \mathbf{X}^{T} \mathbf{Y} \tag{10.2}
\end{equation*}
\]
and MLE of the variance is
\[
\begin{equation*}
\hat{\sigma}^{2}=\frac{(\mathbf{Y}-\mathbf{X} \hat{\beta})^{T}(\mathbf{Y}-\mathbf{X} \hat{\beta})}{N-r_{X}} \tag{10.3}
\end{equation*}
\]
where \(N\) is the number of observations and \(r_{X}\) is rank of \(\mathbf{X}\) (number of columns of the design matrix).

Standard errors for the \(j\)-th parameter can be obtained as
\[
\begin{equation*}
\text { s.e. }\left(\hat{\beta}_{j}\right)=\hat{\sigma}^{2}\left(\mathbf{X}^{T} \mathbf{X}\right)_{j j}^{-1} \tag{10.4}
\end{equation*}
\]
where \(\left(\mathbf{X}^{T} \mathbf{X}\right)_{j j}^{-1}\) stands for the \(j\)-th diagonal element of the inverse of matrix ( \(\mathbf{X}^{T} \mathbf{X}\) ).

Logistic and survibal to be filled in here...

\subsection*{10.2 Analysis of imputed data with ProbABEL}

ProbABEL takes three files as input: a file containing SNP information (e.g. MLINFO file of MACH), file with genome- or chromosome-wide predictor information (e.g. MLDOSE file of MACH), and a file containing phenotype of interest and covariates.

Optionally, the map information can be supplied (e.g. "legend" files of HapMap).

SNP information file
In the simplest scenario, SNP information file is an MLINFO file generated by MACH. This must be a space or tab-delimited file containing SNP name, coding for allele 1 and 2 (e.g. A, T, G or C), frequency of allele 1 , minor allele frequency and two quality metrics ("Quality" = average maximum posterior probability and "Rsq" - ratio between observed and expected variances).

Actually, for ProbABEL, it does not matter what is written in this file - this information is just brought forward to the output. However, it is critical that the number of columns is seven and the number of lines in the file is equal to the number of SNPs in the corresponding DOSE file (plus one for the header line).

The example of SNP information file content follows here (also to be found in ProbABEL/example/test.mlinfo)

Note that header line is present in the file. The file describes five SNPs.
Genomic predictor file
Again under simplest scenario this is a MLDOSE file generated by MACH. Such file starts with two special columns plus, for each of the SNPs under consideration, a column containing the estimated allele 1 dose. The first "special" column is made of the sequential id, followed by an arrow followed by study ID (the one specified in MACH input files). The second column contains method (e.g. "MLDOSE") keyword.

An example of the few first lines of an MLDOSE file for five SNPs described in SNP information file follows here (also to be found in ProbABEL/example/test.mldose)

The order of SNPs in the SNP information file and DOSE-file must be the same. This should be the case if you just used MACH outputs.

Thus, by all means, the number of columns in the genomic predictor file must be the same as the number of lines in the SNP information file plus one.

Phenotypic file
Phenotypic data file contains phenotypic data, but also specifies the analysis model. There is a header line, specifying the variable names. The first column should contain personal study IDs. It is assumed that both the total number and the order of these IDs is are exactly the same as in the genomic predictor (MLDOSE) file described in previous section. This is not difficult to arrange using e.g. R; example is given in ProbABEL/examples directory.

Missing data should be coded with 'NA', 'N' or 'NaN' codes. Any other coding will be converted to some number which will be used in analysis! E.g. coding missing as '-999.9' will result in analysis which will consider -999.9 as indeed true measurements of the trait/covariates.

In case of linear or logistic regression (programs palinear and palogist, respectively), the second column specifies the trait under analysis, while the third, fourth, etc. provide information on covariates to be included into analysis. An example few lines of phenotypic information file designed for linear regression analysis follow here (also to be found in ProbABEL/example/height.txt)

Note again that the order of IDs is the same between MLDOSE and phenotypic data file. The model specified by this file is height \(\sim \mu+s e x+a g e\), where \(\mu\) is intercept.

Clearly, you can for example include sex x age interaction terms by specifying another column having a product of sex and age here.

For logistic regression, it is assumed that in the second column cases are coded as " 1 " and controls as " 0 ". An example few lines of phenotypic information file designed for logistic regression analysis follow here (also to be found in ProbABEL/example/logist_data.txt)

You can see that in the first 10 people, there are three cases, as indicated by "chd" equal to one. The model specified by this file is chd \(\sim \mu+s e x+a g e+\) othercov.

In case of Cox proportional hazards model, the composition of the phenotypic input file is a bit different. In the second column and third column, you need to specify the outcome in terms of follow-up time (column two) and event (column three, " 1 " if event occurred and zero if censoring). Columns from four inclusive specify covariates to be included into analysis. An example few lines of
phenotypic information file designed for Cox proportional hazards model analysis follow here (also to be found in ProbABEL/example/coxph_data.txt)

You can see that for first 10 people, event happens for three of them, while for the other seven there is no event during follow-up time, as indicated by "chd" column. Follow-up time is specified in the preceding column. The covariates included into the model are age (presumably at baseline), sex and "othercov"; thus the model, in terms of R/survival is \(\operatorname{Surv}(\) fuptime_chd, chd \() \sim\) sex + age + othercov.

Optional map file
If you would like that map information (e.g. base pair position) to be included in your outputs, you can supply a map file. These follow HapMap "legend" file format. For example, for the five SNPs we considered the map-file may look like

The order of the SNPs in the map file should follow that in the SNP information file. Only information from the second column - the SNP location - is actually used to generate the output.

\section*{Running analysis}

To run linear regression, you should use program called palinear; for logistic analysis use palogist, and for Cox proportional hazards model use pacoxph (to be found in ProbABEL/bin/ directory after you have compiled the program).

There are in total 11 command line options you can specify to ProbABEL analysis functions linear or logistic. If you run either program without any argument, you will get a short explanation to command line options:
```

user@server~\$ palogist

```

Usage: ../bin/palogist options
Options:
\[
\begin{aligned}
& \text {--pheno : phenotype file name } \\
& \text {--info : information (e.g. MLINFO) file name } \\
& \text {--dose : predictor (e.g. MLDOSE) file name } \\
& \text {--map : [optional] map file name } \\
& \text {--nids : [optional] number of people to analyse } \\
& \text {--chrom : [optional] chromosome (to be passed to } \\
& \text { output) } \\
& \text {--out : [optional] output file name (default is } \\
& \text { regression.out.csv) } \\
& \text {--skipd : [optional] how many columns to skip in } \\
& \text { predictor (dose) file (default 2) } \\
& \text {--ntraits : [optional] how many traits are analysed } \\
& \text { (default 1) } \\
& \text {--separat : [optional] character to separate fields } \\
& \text { (default is space) } \\
& \text {--no-head : do not report header line } \\
& \text {--help : print help }
\end{aligned}
\]

However, for a simple run you can use only three, which specify the necessary files needed to run regression analysis.

These options are -dose (or -d), specifying genomic predictor / MLDOSE file described in sub-section 10.2 -pheno (or -p ), specifying the phenotypic
data file described in sub-section 10.2 and -info (or -i), specifying the SNP information file described in sub-section 10.2

If you change to the ProbABEL/example directory you can run analysis of height by
```

user@server~/ProbABEL/example/\$ ../bin/palinear -p height.txt

```
    -d test.mldose -i test.mlinfo

Output from analysis will be directed to "regression.out.csv" file. You can run analysis of binary trait "chd" with
```

user@server~/ProbABEL/example/\$ ../bin/palogist -p logist_data.txt
-d test.mldose -i test.mlinfo

```

To run a Cox proportional hazards model, try
user@server~/ProbABEL/example/\$ ../bin/pacoxph -p coxph_data.txt
-d test.mldose -i test.mlinfo
Please have a look at the shell script files example_qt.sh, example_bt.sh and example_all.sh to have a better overview of analysis options.

\section*{Output file format}

Let us consider what comes out of the linear regression analysis described in the above section. After you have run the analysis, in the output file you will find something like

Here, I show only three first lines of output. Note that lines starting with "..." are actually the ones continuing the previous line - I just have wrapped this output so we can see these long lines.

The header provides short description of what can be found in the specific column. The first column provides the SNP name and next six are descriptives which were brought directly from the SNP information file. Thus these describe allele frequencies and quality in your total imputations, not necessarily in the data under analysis.

On the contrast, starting with the next column, named "n", the output concerns the data analysed. Column 8 ("n") tells the number of subjects for whom complete phenotypic information was available. At this point, unless you have complete measurements on all subjects, you should feel warned if the number here is exactly the number of people in the file - this probably indicates you did not code missing values according to ProbABEL format ('NA', 'NaN', or ' N ').

The next column nine ("Mean_predictor/2") gives you the mean value of the predictor (SNP) divided by 2 . When MLDOSE file is used, this corresponds to the estimated frequency of allele 1 in people with complete phenotypic data.

If "-chrom" option was used, in the next column you will find the value specified by this option. If "-map" option was used, in next column you will find map location brought from the map-file. Next columns, starting with "beta_mu", provide estimates for the regression coefficients ("beta_mu" is a bit awkward name for intercept). The number of \(\beta \mathrm{s}\) reported is equal to the number of covariates in the phenotypic file plus two (intercept and \(\beta_{S N P}\) ). It is likely that you will be most interested in "beta_SNP" - the estimate of the regression coefficient for the SNP in consideration. Next columns provide corresponding
standard errors of the estimates ("sebeta_mu", ... "sebeta_SNP"). Column names "sigma2" provides residual variance; finally, "SNP_Z" provides the value of the Wald's test computed as
\[
Z_{S N P}=\frac{\beta_{S N P}}{s . e . \beta_{S N P}}
\]
while "SNP_chi2" provides the \(\chi_{1}^{2}\) test obtained as square of \(Z_{S N P}\).

\section*{Preparing input files}

In the ProbABEL/bin directory you can find perepare_data. \(R\) file - an \(R\) script which arranges phenotypic data in right format. Please read this script for details.

Memory use and performance
Maximum likelihood regression is implemented in ProbABEL. With 6,000 people and 2.5 millions SNPs, genome-wide scan is completed in less that an hour for linear model with 1-2 covariates and overnight for logistic regression or Cox proportional hazards model.

Memory is an issue with ProbABEL - large chromosomes, such as chromosme one consumed up to 5 Gb RAM with 6,000 people.

\section*{Chapter 11}

\section*{Meta-analysis of GWA scans}

\subsection*{11.1 Standard meta-analysis methods}

Imagine you are interested in the effect of a certain polymorphism onto a particular disease. After scanning literature, you find some studies that implicate certain allele as significantly increasing the risk of the disease, but you will typically find also that other studies were inconclusive (no significant association), and that even some of the studies implicated the same allele as "protective". Your gut feeling may be that the allele is indeed the risk one, because you feel that the studies contradicting to this hypothesis were based on small number of subjects; however, how do you quantify this feeling? In this situation you need to perform meta-analysis of available data to come up with the joint effect size estimate and P -value, as based on all available data.

Let us first consider a situation when you are interested in the effect of the allele on a quantitative phenotype, expressed as a coefficient of regression of the trait onto the number of this allele in the genotype. Under a favourable scenario, from every individual study you would know the estimate of this regression coefficient, and the standard error of the estimate (or, equivalently, the \(P\)-value or the test statistics value for association).

One of approaches frequently used in meta-analysis of the data coming from a number of independent studies is the inverse variance method. In essence, this method is equivalent to combining likelihoods coming from separate studies, using quadratic approximation. Denote coefficients of regression estimated in \(N\) studies as \(\beta_{i}\), and associated squared standard errors of the estimates as \(s_{i}^{2}\) where \(i \in 1,2, \ldots, N\). Note that the regression coefficient should be reported on the same scale, e.g. centimeters, meters, or using observations reported on the standard normal scale. Define weights for individual studies as
\[
w_{i}=\frac{1}{s_{i}^{2}}
\]

Then the pooled estimate of the regression coefficient is
\[
\beta=\frac{\sum_{i=1}^{N} w_{i} \beta_{i}}{\sum_{i=1}^{N} w_{i}}
\]

As you can see, the weights have straightforward interpretation: the bigger the weight of the study (meaning the small is the standard error in the study), the larger is the contribution from this study onto the pooled estimate.

The standard error of the pooled estimate is computed as
\[
s^{2}=\frac{1}{\sum_{i=1}^{N} w_{i}}
\]
and the \(\chi^{2}\)-test for association is computed in standard manner as
\[
T^{2}=\frac{\beta^{2}}{s^{2}}=\frac{\left(\sum_{i=1}^{N} w_{i} \beta_{i}\right)^{2}}{\sum_{i=1}^{N} w_{i}}
\]
or, alternatively, the Z-test is
\[
Z=\frac{\beta}{s}=\frac{\sum_{i=1}^{N} w_{i} \beta_{i}}{\sqrt{\sum_{i=1}^{N} w_{i}}}
\]

Let us try to do meta-analysis using the inverse variance pooling method. Imagine we have information from four different studies reporting effect and the standard error of the same allele:

Table 11.1: Estimated regression coefficients from four studies
\begin{tabular}{lcccc}
\hline Study & \(n\) & \(\beta\) & \(s_{\beta}\) & \(\chi^{2}\) \\
\hline 1 & 225 & 0.16 & 0.07 & 5.224 \\
2 & 560 & 0.091 & 0.042 & 4.694 \\
3 & 437 & 0.072 & 0.048 & 2.25 \\
4 & 89 & -0.03 & 0.12 & 0.062 \\
\hline Total & 1311 & \(?\) & \(?\) & \(?\) \\
\hline
\end{tabular}

Let us try to access the joint significance of the association using these data. First, let us define a vector of regression coefficients and squared standard errors:
```

> beta <- c(0.16, 0.091, 0.072, -0.03)
> s <- c(0.07, 0.042, 0.048, 0.12)
> s2 <- s * s
> s2

```
[1] 0.0049000 .0017640 .0023040 .014400
Compute the weight for individual studies as
```

> w <- 1/s2
> w

```
[1] \(204.08163566 .89342434 .02778 \quad 69.44444\)
Estimate pooled regression coefficient as
```

> pbeta <- sum(w * beta)/sum(w)
> pbeta

```
[1] 0.08898527
and pooled squared standard error as
```

> ps2<- 1/sum(w)
> ps2

```
[1] 0.0007846539
To access significance of association in meta-analysis, let us compute \(\chi^{2}\) test statistics and the \(P-\) value with
```

> pchi2 <- pbeta * pbeta/ps2

```
> pchi2
[1] 10.09155
```

> ppvalue <- 1 - pchisq(pchi2, 1)

```
> ppvalue
[1] 0.001489504
We conclude that there is a significant association in meta-analysis.
There is an important effect which should be considered when doing metaanalysis of published data. Given numerous polymorphisms available in human genome, a particular polymorphism usually becomes a focus of interest only when it was shown to be significantly associated in some study which reports it. Put it other way around: only when a significant association was detected and reported, more studies are likely to be performed on the same polymorphism. This first report, however, is not guaranteed to demonstrate a true association: it may well report a false-positive, or, even in presence of association, it is likely to over-estimate the effect of the polymorphism. Thus there is a positive bias in literature reports, and this bias is particularly strong for the first report, a phenomenon frequently referenced to as "champion's" or "winner's curse".

The observations we have just considered are quite typical in that the first study, where the association was originally discovered, reports the biggest effect and most significant effect, while the follow-up studies suggest smaller effect.

Therefore, when you meta-analyse data from publications it is always good idea to exclude the first report (in case it is positive - and it is always positive!) and check if significant association is still observed. Let us try to do that:
```

> beta <- beta[2:4]
> s2 <- s2[2:4]
> w <- w[2:4]
> pbeta <- sum(w * beta)/sum(w)
> pbeta
[1] 0.07544522
> ps2 <- 1/sum(w)
> ps2
[1] 0.0009342602

```
```

> pchi2 <- pbeta * pbeta/ps2
> pchi2

```
[1] 6.092501
```

> ppvalue <- 1 - pchisq(pchi2, 1)
> ppvalue

```

\section*{[1] 0.01357568}

Indeed, when the first "champion" report is excluded, the overall evidence is decreased and results become less significant, though still pointing to the same direction.

When binary traits are studied, and results are reported as Odds Ratios with \(P-\) values, it is also possible to apply inverse variance method. For this, you need to transform your Odds Ratios using natural logarithm, and, on this scale, estimate the standard error. Generic inverse variance pooling may be applied to the data transformed this way; the final results are back-transformed onto Odds Ratio scale using exponentiation.

Let us consider a simple example. Let Odds Ratios and \(\chi^{2}\) test statistics values coming from four studies of a binary phenotype are as following: \(\theta_{1}=1.5\) \(\left(\chi^{2}=5.1\right), \theta_{2}=1.3\left(\chi^{2}=2.2\right), \theta_{3}=0.9\left(\chi^{2}=0.5\right), \theta_{4}=1.2\left(\chi^{2}=3.1\right)\).

Let us first transform the Odds Ratio to the logarithmic scale with
```

> or <- c(1.5, 1.3, 0.9, 1.2)
> lnor <- log(or)
> lnor
[1] $0.4054651 \quad 0.2623643-0.1053605 \quad 0.1823216$

```

To compute standard errors from known \(\chi^{2}\) values, one can use simple relation
\[
\chi^{2}=\frac{\beta^{2}}{s^{2}}
\]
and thus
\[
s^{2}=\frac{\beta^{2}}{\chi^{2}}
\]

Thus to compute the square standard errors corresponding to the log-Odds Ratio, we can use
```

> chi2or <- c(5.1, 2.2, 0.5, 3.1)
> s2lnor <- lnor * lnor/chi2or
> s2lnor

```

\section*{[1] 0.032235680 .031288640 .022201680 .01072295}

Now we can combine log-Odds Ratios and corresponding standard errors using inverse variance method:
```

> w <- 1/s2lnor
> plnor <- sum(w * lnor)/sum(w)
> plnor

```
```

[1] 0.1650462

```
```

> ps2 <- 1/sum(w)

```
> ps2 <- 1/sum(w)
> ps2
[1] 0.004968165
```

> pchi2 <- plnor * plnor/ps2
> pchi2
[1] 5.482958

```
> ppval <- 1 - pchisq(pchi2, 1)
```

> ppval
[1] 0.01920274
And the corresponding estimate of pooled Odds Ratio is

```
> exp(plnor)
```

[1] 1.179448
and $95 \%$ confidence interval is

```
> exp(plnor - 1.96 * sqrt(ps2))
```

[1] 1.02726
$>\exp (p l n o r+1.96 * \operatorname{sqrt}(p s 2))$

## [1] 1.354181

Some times, effects are reported on different scales, and/or there may be suspect that these effects are not translatable across studies because of the differences in experimental design or for some other reasons. In this case, it may be better to poll the data without use of the effect estimate in exact manner, based only on the sign of association and its significance. This can be done by pooling Z-score values. Z-score refers to the test statistics, which has standard normal distribution under the null and can be derived e.g. by dividing estimate of the regression coefficient onto its standard error:

$$
Z_{i}=\frac{\beta_{i}}{s_{i}}
$$

The Z-score pooling methods can be derived from the inverse variance pooling by exploiting the fact that generally standard error of the estimate is proportional to $1 / \sqrt{n}$, where $n$ is the number of observations used for estimation. Therefore individual scores are assigned weights which are proportional to the square root of number of independent observations used in individual study, $w_{i}=\sqrt{n_{i}}$. The pooled Z-score statistics is computed as

$$
Z=\frac{\sum_{i=0}^{N} w_{i} Z_{i}}{\sqrt{\sum_{i=0}^{N} w_{i}^{2}}}
$$

We can now repeat the analysis of our first data set using Z-score pooling method. First, our data from table 11.1 are

```
>n <- c(225, 560, 437, 89)
> beta <- c(0.16, 0.091, 0.072, -0.03)
> s <- c(0.07, 0.042, 0.048, 0.12)
```

The Z-scores and weights are are:

```
> z <- beta/s
> z
[1] 2.285714 2.166667 1.500000 -0.250000
> W <- sqrt(n)
> W
```

[1] $15.000000 \quad 23.664319 \quad 20.904545 \quad 9.433981$

The pooled estimate of Z-score is

```
> pz<- sum(w * z)/sqrt(sum(w^2))
> pz
```

[1] 3.163875
and corresponding $P-$ value is

```
> 1 - pchisq(pz * pz, 1)
```

[1] 0.001556839
which is almost the same $P$-value we have obtained previously using the inverse variance method. Note, however, that now we do not know the "pooled" estimate of the regression coefficient.

Other important aspects of meta-analysis, such as heterogeneity, and a wide range of methods different from the inverse variance and Z-score based methods are not covered here, and we refer the reader to more epidemiologically-oriented literature for a better review.

### 11.2 Exercise: meta-analysis of literature data

In this exercise, you will perform meta-analysis of results collected from literature. These results resemble these obtained for association analysis between Pro12Ala polymorphism of the PPAR- $\gamma$ gene and type 2 diabetes. The data collected from literature are presented in the table 11.2 .

As you can see, only the original study report significant association, while other four are insignificant and one point in opposite direction.

Answer the following questions:
Exercise 37 Perform meta-analysis of the data presented in table 11.2 . Which allele is the risk one? Is this risk significant? What is pooled Odds Ratio and $95 \%$ confidence interval? Do analysis using at least two methods. Which method is better (best) in this situation? Why?

Exercise 38 Perform meta-analsys excluding the original report (study 1). Is there still significant association between Pro12Ala and diabetes?

Table 11.2: Summary of six studies of association between T2D and Pro12Ala polymorphism of the PPAR- $\gamma$ gene. $n$ : number of subjects; effective allele: the allele for which the OR was estimated.

| Study | Effective allele | $n$ | $O R_{E}$ | $P-$ value |
| :--- | :---: | :---: | :---: | :---: |
| 1 | Ala | 221 | 0.67 | 0.013 |
| 2 | Pro | 306 | 0.93 | 0.60 |
| 3 | Pro | 71 | 1.08 | 0.84 |
| 4 | Ala | 164 | 0.83 | 0.40 |
| 5 | Pro | 242 | 1.22 | 0.25 |
| 6 | Pro | 471 | 1.23 | 0.07 |

### 11.3 Reporting GWA results for future metaanalysis

In this section, we will discuss specifics of GWA analysis when meta-analysis is aimed at later stage. In order to perform meta-analysis at later stage, using either inverse variance or Z-score based method, you generally need to report only effect estimates, standard errors of The estimates (or, equivalently, $P-$ values or test statistics values), and number of observations used for estimation.

It is also clear that it is crucial to know for which allele the effect is reported, and this is the point where meta-analysis of genetic data may be very confusing. Generally, one may think that reporting what couple of nucleotide bases correspond to the polymorphism under the study and defining what allele was used as reference in the regression model may be enough. This, however, is not true for certain class of polymorphisms and may be a source of great confusion.

Consider a DNA molecule; as you know it is made of two complementary strands (forward or " + " and reverse or "-"). As you may guess, depending on the strand, what is an "A/G" polymorphism when reported on " + " strand becomes "T/C" polymorphism on the "-" strand (using complementarity rule $\mathrm{A}<->\mathrm{T}$ and $\mathrm{G}<->\mathrm{C}$ ). This is not a big problem for most of the polymorphism classes, because if say you know that for a first study $\beta_{1}$ is reported for the " G " allele of the "A/G" polymorphism and in the second study $\beta_{2}$ is the estimate of the effect "C" allele of the same polymorphism, but coded as "T/C" (thus other strand), you can easily spot that and say the "C" is the same as " G " in this situation, and pool the two betas straightforwardly.

However, for two types of polymorphisms, "A/T" and "G/C", where you can not get away without knowing what the strand was: what is reported as the effect of " $T$ " in "A/T" polymorphism in study one; and seemingly corresponding effect of "T" in "A/T" polymorphism in study two may be apparently reports for two opposite alleles, if strands used for reporting in two studies were different.

The story may become even more complex, because the forward/reverse orientation depends on the genomic build ${ }^{1}$

Thus if you want to pool your results with the results of others, there are quite a few SNP characteristics which are absolutely crucial to report, namely, the nucleotide bases describing the polymorphism, with indication which one was used as reference, and which one as "effective", strand, genomic build, and only

[^6]than the effect estimate, standard error of the effect, and number of observations used to do estimation.

Other characteristics which are also recommended for reporting because they describe the quality characteristics of the sample and/or provide redundant information, which is good for double checks. Such characteristics include: frequency of the effective or reference allele, call rate, P-value for Hardy-Weinberg equlibrium and may be some parameter describing what is the direction of deviation from HWE (e.g. $F_{\max }$ ). When reporting results for imputed SNPs, more quality control characteristics should be included, suh as average maximal posterior probability, $R^{2}$, etc.

Let us start with arranging two data sets which could then be used for metaanalysis. Basically, we will use cleaned data from the GWA exercise you did in section 5 (5) page 69), and split that is two parts.

Let us first load the data and re-name the data object:

```
> data(ge03d2ex.clean)
> data2 <- ge03d2ex.clean
> rm(ge03d2.clean)
```

and then split it in two parts:

```
> data2@gtdata@nids
```

[1] 125

```
mdta1 <- data2[1:40, ]
mdta2 <- data2[41:data2@gtdata@nids, ]
```

We will analyse body mass index. If you pooling results of analysis of studies which are designed in approximately the same manner, you may think of reporting the effect estimates on the same scale and use of the inverse variance method for meta-analysis.

However, in meta-analysis of multiple data sets different individual studies are likely to assess different population, will use different designs, measure different covariates, and so on. Therefore you should think of some standardisation of the outcome variable (or apply Z-score method).

Therefore for the purpose of future meta-analysis, it becomes conventional to analyse pre-adjusted data which are scaled to Standard Normal (mean of zero and variance of unity). Note that this argument applies only to meta-analysis you may and should report effects on the original scale (e.g. in centimeters and grams) in analysis of individual studies, in order to have better interpretability.

Moreover, in meta-analysis you heavily rely on the large numbers approximation when estimating $P$-values; while for individual study you can always apply empirical, e.g. permutation-based, procedures to derive the correct $P$-value whatever is the distribution of the trait, in meta-analysis the Normality assumption becomes crucial, and you do not want few outliers spoiling your results by screwing up $P$-values. Therefore some transformation improving normality is desirable. Note that transformation to Standard Normal does not improve the fit to normality; to do that other transformation should be applied. Probably the most famous transformations are log- and square root ones, then one may think of Box-Cox transformation. At the same time there is a transformation,
called a Rank Transformation ro Normality which guarantees perfect fit to Normal in absence of heavy ties $\xi^{2}$ We advocate the use of Rank Transformation to Normal for meta-analysis purposes.

GenABEL implements the ztransform function for the purposes of Z-transformation. This function takes two (actually three - see help for details) arguments: formula (or just the variable name) and data. ztransform function will perform (generalised) linear regression using the specified formula, and will transform the residuals from analysis onto Z-scale by subtracting the mean and division by the standard deviation.

Let us consider what this function does practically. Let us first transform BMI from the first set without using covariates:

```
> zbmiO <- ztransform(bmi, mdta1)
```

The histogram of the transformed variable and scatter-plot of raw against transformed BMI is given at figure 11.1, column 1. Note that the fit to Normality is not improved by this transformation; with the original BMI, the Shapiro test for deviation from normality gives

```
> shapiro.test(mdta1@phdata$bmi)
    Shapiro-Wilk normality test
data: mdta1@phdata$bmi
W = 0.9328, p-value = 0.01990
```

with identical results from the transformed variable:

```
> shapiro.test(zbmi0)
    Shapiro-Wilk normality test
data: zbmiO
W = 0.9328, p-value = 0.01990
```

This is quite natural: as you can note from scatter-plot in column 1 of figure 11.1. only the centering and the spread of the scales are different for X (original $\overline{\mathrm{BMI}}$ ) and $\mathrm{Y}(\mathrm{x} 0)$, otherwise there is an exact linear correspondence between the two.

We can also do transformation using sex and age-adjusted residuals with

```
> zbmi1 <- ztransform(bmi ~ sex + age, mdta1)
```

The scatter-plot of raw against transformed BMI is given at figure 11.1, column 2. Note that this transformation may slightly change the fit to Normal, which happens because we factor out the effects of sex and age:

```
> shapiro.test(zbmi1)
    Shapiro-Wilk normality test
data: zbmi1
W = 0.9263, p-value = 0.01224
```

[^7]From the scatter-plot in column 2 of figure 11.1 , it is quite clear what happens: the residuals from linear regression are not corresponding to the original BMI in exact linear manner.

A similar function, which performs rank-transformation to normality, is named rntransform. For example if we want to adjust BMI for sex and age and rank-transform the residuals to Normal, we can use

```
> rnbmi1 <- rntransform(bmi ~ sex + age, mdta1)
```

This transformation, however, indeed improves the fit to Normal:

```
> shapiro.test(rnbmi1)
    Shapiro-Wilk normality test
data: rnbmi1
W = 0.999, p-value = 1
```

In essence, the $P$-value of 1 means perfect fit to Normal - and this is what should have occurred when this transformation is used on the data without ties. Perfectly Normal distribution of the transformed trait may be enjoyed at the histogram presented at column 3 of figure 11.1 .

Let us analyse Rank-Normal-transformed, sex and age-adjusted BMI in the two data sets, using qtscore function. Analysis of the first study is done with

```
> qts1 <- qtscore(rnbmi1, mdta1)
```

and analysis of the second study is done with

```
> zbmi2 <- ztransform(bmi ~ sex + age, mdta2)
> qts2 <- qtscore(zbmi2, mdta2)
```

The analysis looks very simple - is not it? However, the real difficulty did not start yet: now we need to extract coding, reference allele, strand, etc. otherwise we can not do right meta-analysis later on!

Let us assume that you want to summarise the GW results from additive 1 d.f. test using following variables: (1) SNP name (2) chromosome (3) position (4) number of people with data available for this SNP test (5) effect of the allele (6) standard error of the effect (7) P -value for the test (8) corrected P -value (we will use Genomic Control) (9) coding, with reference allele coming first (10) strand (11) frequency of the reference allele and, finally, (11) $P$-value from the exact test for HWE (that would be good for QC checks later on).

In the above list of output parameters, two are not directly available - frequency of the reference allele, which can be computed with

```
> refallfreq <- 1 - summary(mdta1@gtdata)$Q.2
```

and the standard error of the effect estimate. Though GenABEL does not report the s.e., it can be computed from the $\chi^{2}$ value of the test. By definition,

$$
\left(\frac{\beta}{S E_{\beta}}\right)^{2}=\chi_{1}^{2}
$$



Figure 11.1: Histogram of transformed BMI and scatter-plots of the raw BMI against transformed BMI. Column 1: Z-transformation without covariates. Column 2: Z-transformation with adjustment for age and sex. Column 3: Ranktransformation to normality, after adjustment for sex and age.
and thus given value of the $\chi^{2}$ test and knowing the effect, we can compute standard error with

$$
S E_{\beta}=\frac{\beta}{\sqrt{\chi^{2}}}
$$

Thus, s.e. is

```
> seeff <- qts1$effB/sqrt(qts1$chi2.1df)
```

At this moment we can arrange the required data frame:

```
> mdf1 <- data.frame(name = qts1$snpnames, chrom = qts1$chromosome,
+ pos = qts1$map, n = qts1$N, beta = qts1$effB, sebeta = seeff,
+ p = qts1$P1df, pgc = qts1$Pc1df, coding = as.character(mdta1@gtdata@coding),
+ strand = as.character(mdta1@gtdata@strand), reffreq = refallfreq)
```

Let us inspect the first 10 raws of the resulting output:

```
>mdf1[1:5,]
```

|  | name chrom | pos | n | beta | sebeta | p | pgc |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| rs7435137 | rs7435137 | 1 | 4259040 | 39 | 0.05838361 | 0.2141932 | 0.7851803 | 0.7851803 |
| rs664063 | rs664063 | 2 | 7288020 | 40 | 0.01829962 | 0.4705679 | 0.9689794 | 0.9689794 |

```
rs546570 rs546570 2 6120257 40 0.27341721 0.3890643 0.4822079 0.4822079
rs7908680 rs7908680 1 2311762 40 0.67913083 0.5908536 0.2503885 0.2503885
rs166732 rs166732 1 4716343 39 0.35418295 0.7213534 0.6234280 0.6234280
rs7435137 coding strand reffreq
rs664063 GC - 0.9375000
rs546570 TA + 0.1000000
rs7908680 CA - 0.9625000
rs166732 TG - 0.9743590
```

However, it is not recommended that you perform above-described reporting actions unless you develop your own format. In case if you plan to use MetABEL for meta-analysis, you best use formetascore function, which basically performs operations similar to described, and reports results in format compatible with MetABEL.

Thus, if you plan to use MetABEL for meta-analysis, required tables can be generated with single command:

```
> mdf1 <- formetascore(bmi ~ sex + age, mdta1, transform = rntransform)
```

You can see that results are the same as previously:

```
> mdf1[1:5, ]
```


rs7435137 0.975
rs664063 1.000
rs546570 1.000
rs7908680 1.000
rs166732 0.975

To write all the data to a file, we can use standard $R$ write.csv function:

```
> write.csv(mdf1, file = "RData/part1.rnbmisexage.csv", row.names = F)
```

Similar analysis is applied to the second data set:

```
> mdf2 <- formetascore(bmi ~ sex + age, mdta2, transform = rntransform)
```

We can inspect the first five lines of the output with

```
> mdf2[1:5, ]
```



Let us write the data to a file:

```
> write.csv(mdf2, file = "RData/part2.rnbmisexage.csv", row.names = F)
```

Finally let us analyse and save results for another data set, ge03d2c:

```
> data(ge03d2c)
> mdf3 <- formetascore(bmi ~ sex + age, ge03d2c, transform = rntransform)
> write.csv(mdf3, file = "RData/part3.rnbmisexage.csv", row.names = F)
```


### 11.4 Meta-analysis with MetABEL

Now we are ready to meta-analyse GWA data coming from three studies. For this we will need to use MetABEL package, implementing simple meta-analysis functions for GWA data. Start with loading the package:

```
> library(MetABEL)
```

We will first meta-analyse the three studies using the data frames generated in previous section, $\operatorname{mdf} 1, \operatorname{mdf} 2$ and $\operatorname{mdf} 3$. For this we will use the core function of MetABEL, metagwa.tables. This function takes four arguments: two data frames with results from individual studies, and two arguments supplying the study names. Pooling of multiple studies is possible by sequential application of this function.

Let us pool two first data frames:

```
> pooled <- metagwa.tables(mdf1, mdf2, name.x = "Part1", name.y = "Part2")
analysing ...
Lambda Part1 = 0.9440436
Lambda Part2 = 1.090242
Corrected Lambda Part1 = 0.9440436
Corrected Lambda Part2 = 1
Lambda POOLED data = 0.9997559
... DONE
```

The pooled data frame contains results of meta-analysis and essential details of the original studies:

```
> pooled[1:5, ]
```

| 1 | rs100616 | - | G C | C | chromosome | position $\quad$ n | npops |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | rs1006497 | + | T G | G | 1 | 2658810123 | 2 |
| 3 | rs1011580 | + | A G | G | 3 | 10048771122 | 2 |
| 4 | rs1011953 | + | A G | G | 2 | 6464510124 | 2 |
| 5 | rs1013473 | + | A $\quad$ T | T | 1 | 4487262124 | 2 |
|  | beta | sebeta | effallelefreq | q call | 1 pexhwe | e obetaPart1 |  |
| 1 | 0.11128490 | 0.1842895 | 0.1260163 | 630.9919667 | 474.7183287 | 70.08819818 |  |
| 2 | -0.07530695 | 0.1933475 | 0.1463415 | 150.9920732 | 320.1698876 | -0.25425751 |  |
| 3 | -0.06827202 | 0.1394457 | 0.5040984 | 840.9839091 | 313552114 | 4-0.06755996 |  |
| 4 | 0.07615424 | 0.1367472 | 0.3306452 | 521.0000000 | 0.1037354 | -0.18982068 |  |
| 5 | 0.30191078 | 0.1318863 | 0.5161290 | 1.0000000 | 0.0000000 | 0.31522388 |  |
|  | obetaPart2 | osePart1 | osePart2 | chi2 | p |  |  |
| 1 | 0.118757384 | 0.3726827 | 0.21202670. | 0.36464570. | . 54593737 |  |  |
| 2 | 0.003368901 | 0.3498751 | 0.23198890. | 0.15170260. | 0.69691357 |  |  |
| 3 | -0.068805418 | 0.2130800 | 0.18442170. | 0.23970410. | . 62441989 |  |  |
| 4 | 0.007546243 | 0.2228918 | 0.17316710. | 0.31013520. | . 57759723 |  |  |
| 5 | 0.295031422 | 0.2259542 | 0.16242575. | 5.24031980. | . 02206922 |  |  |

If one needs to pool more studies, this data frame should be used as the first argument of the metagwa.tables, and name.x argument should take special value "POOLED":

```
> pooled <- metagwa.tables(pooled, mdf3, name.x = "POOLED", name.y = "mdf3")
NA for betas in both populaions
1 8 \text { SNPs removed}
analysing ...
Lambda mdf3 = 1.128096
Corrected Lambda mdf3 = 1
Lambda POOLED data = 1.357809
... DONE
> pooled[1:5, ]
```



|  | obetaPart2 | obetamdf3 | osePart1 | osePart2 | osemdf3 | chi2 | p |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | NA | -0.055213494 | NA | NA | 0.11185255 | 0.24366808 | 0.6215693 |
| 2 | NA | -0.085048708 | NA | NA | 0.13218866 | 0.41394922 | 0.5199718 |
| 3 | NA | -0.037120652 | NA | NA | 0.08523891 | 0.18965112 | 0.6632071 |
| 4 | 0.118757384 | 0.121082405 | 0.3726827 | 0.2120267 | 0.15961076 | 0.93852071 | 0.3326586 |
| 5 | 0.003368901 | 0.005382408 | 0.3498751 | 0.2319889 | 0.14107317 | 0.03950226 | 0.8424569 |

This procedure may become quite laborious if multiple studies are to be pooled. In this case, it is possible to run meta-analysis using data provided in files, by applying function metagwa.files. As the first argument, this function takes the path to the directory where the files with results of individual studies are stored. It is assumed that the file names are made of two parts: population/study name and an extension. Thus the second argument of the metagwa.files function is the vector with names of studies, and the third one provides extension. Other arguments, "maf", "call" and "phwe" provide the threshold for QC filtering of SNPs in individual studies

The function does not return any value, but rather creates a new file named POOLEDextens, where "extens" is the argument supplied to the function, in the source directory. To run analysis on the three files in the directory "RData" we can use

```
> metagwa.files(dir = "RData", pops = c("part1", "part2", "part3"),
+ extens = ".rnbmisexage.csv", maf = 1e-04, call = 0.93, phwe = 1e-08)
Population part1 , reading RData/part1.rnbmisexage.csv done
Dimesions after filters are 3678 15
population part2, reading RData/part2.rnbmisexage.csv done
Dimesions after filters are 3823 15
analysing ...
Lambda part1 = 0.9403026
Lambda part2 = 1.090242
Corrected Lambda part1 = 0.9403026
Corrected Lambda part2 = 1
Lambda POOLED data = 0.9971353
... DONE
Dimesions after pooling are 3868 20
population part3, reading RData/part3.rnbmisexage.csv done
Dimesions after filters are 7493 15
analysing ...
Lambda part3 = 1.127385
Corrected Lambda part3 = 1
Lambda POOLED data = 1.360099
... DONE
Dimesions after pooling are 7493 22
$analysed.pops
[1] "part1" "part2" "part3"
> poolf <- read.csv("RData/POOLED.rnbmisexage.csv", strings = F)
> poolf[1:5, ]
    name strand allele1 allele2 effallele chromosome position n npops
```




## Chapter 12

## Analysis of selected region

### 12.1 Exploring linkage disequilibrium

See help for r2fast.

### 12.2 Haplotype analysis

Use
> gtforld <- as.hsgeno(srdta[, 1:5])
to convert part of your SNPs to haplo.stats format.
You can also use interface function to do sliding widow analysis
> h2 <- scan.haplo("qt1 ~CRSNP", srdta, snps = c(1:5))

### 12.3 Analysis of interactions

See help for scan.haplo.2D and scan.glm.2D

## Appendix A

## Importing data to GenABEL

As described in section 4.1, GenABEL gwaa.data-class consist of phenotypic data frame and an object of snp.data-class, which contains all genetic data. To import data to GenABEL, you need to prepare two files: one containing the phenotypic, and the other containing genotypic data.

The phenotype file relates study subject IDs with values of covariates and outcomes. In the phenotypic data file, the first line gives a description (variable name) of the data contained in a particular column; the names should better be unique, otherwise R will change them.

The first column of the phenotype file must contain the subjects' unique ID, named "id". The IDs listed here, and in the genotypic data file, must be the same. It is recommended that the id names are given in quotation marks (see example below), which will save you a possible troubles with e.g. leading zeros.

There also should also be a column named "sex" and giving sex information ( $0=$ female, $1=$ male $)$. Other columns in the file should contain phenotypic information.

Missing values should be coded with "NA"; binary traits should have values 0 or 1 .

All subjects present in the genotypic files must be listed in the phenotypic file as well, because sex information provided by the phenotypic file is an essential part of the genotypic QC procedure.

An example of few first lines of a phenotype file is as follows:

| id | sex | age | bt1 | qt | qt1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| "cd289982" | 0 | 30.33 | NA | NA | 3.93 |
| "cd325285" | 0 | 36.514 | 1 | 0.49 | 3.61 |
| "cd357273" | 1 | 37.811 | 0 | 1.65 | 5.30 |
| "cd872422" | 1 | 20.393 | 0 | 1.95 | 4.07 |
| "cd1005389" | 1 | 28.21 | 1 | 0.35 | 3.90 |

This file tells us that, for example, person 325286 is female ( 0 in second column), and she has " 1 " (usually this means a "case") value for the trait "bt1", so on. Person 289982 has measurements only for sex, age and qt1, while other measurements are missing (NA, Not Available).

If you need to add pehnotypes to a gwaa. data-class object already created, you can use function add.phdata. This function allows you to add variables contained in some data frame to the existing data@phdata object. The data
frame to be added should contain "id" variable, identical to that existing in the object, and should not contain any othe variables with names identical to these already existing.

The second file you need should contains genotypic data. As described in section 4.1 4.1, page 53), GenABEL snp.data-class contains different types of information. For every SNP, information on map position, chromosome, and strand should be provided. For every person, every SNP genotype should be provided. GenABEL provides a number of function to convert these data from different formats to the internal GenABEL raw format. We will first consider our preferred format, which we informally call "Illumina"-like.

## A. 1 Converting from preferred format

We will consider use of convert.snp.illumina procedure; details of other procedures are given later. Note that what we call "illumina" format is not really a proprietary format from that company, it is just one of the possible text output format from the Illumina BeadStudio; similar formats are accepted/generated by HapMap and Affymetrix.

The file of the "Illumina" format contains SNPs in rows and IDs in columns. The first line is a "header", containing column names. The first three columns should contain information on SNP name, chromosome, and position. There is an optional (though highly recommended!) fourth column, containing strand information (acceptable codes: "+","-","u", the last stands for "unknown"). After that column, each of the residual ones corresponds to an individual, with ID as the column name. Genotypes should be presented by two consecutive characters (no separator).

An example of few first lines of the "illumina" genotypic file is as follows:

| name | chr | pos | strand | "cd289982" | "cd325285" | "cd357273" | "cd872422" |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | "cd1005389"

It is clear that is not quite conventional Illumina file - because in BeadStudio the alleles are reported using the "top" strand; rather, this is an Affymetrix or HapMap-type of a file. Anyways, this file contains all required genotypic information, and this file format is the preferred one for import. Assume that the file with the genotypic data is called "gen0.illu", and is stored in the directory "RData". You can convert the data to GenABEL raw format by

```
> convert.snp.illumina(inf = "RData/gen0.illu", out = "RData/gen0i.raw",
+ strand = "file")
Reading genotypes from file 'RData/gen0.illu' ...
Writing to file 'RData/gen0i.raw' ...
... done.
```

Here is the content of the converted file "gen0i.raw" - internal raw data representation:

```
#GenABEL raw data version 0.1
"cd289982" "cd325285" "cd357273" "cd872422" "cd1005389"
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
1 9 22 X XY Y mt
12352344 3455 32535 2344 23556 24245
04 0c Of 08 Oc 08 Of
010101 02 02 01 02
69 c0
9640
d5 80
6540
9640
0 7 4 0
d0 00
```

Note the option strand="file" - it is telling GenABEL that strand information is provided in the file.

At this moment, you can load the data into GenABEL. Assume that the phenotypic file described above is called "phe0.dat" and the converted genotypic file in the raw GenABEL format is called "gen0i.raw". You can load the data using the command

```
> df <- load.gwaa.data(phe = "RData/pheO.dat", gen = "RData/genOi.raw",
+ force = T)
ids loaded...
marker names loaded...
chromosome data loaded...
map data loaded...
allele coding data loaded...
strand data loaded...
genotype data loaded...
snp.data object created...
assignment of gwaa.data object FORCED; X-errors were not checked!
```

The option "force=TRUE" tells that GenABEL should load the data even if it fins sex errors.

You can inspect the loaded data; let us first look into phenotypic data by by

```
> df@phdata
```

|  | id |  | sex | age | bt1 | qt |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: |
| qt1 |  |  |  |  |  |  |
| cd289982 | cd289982 | 0 | 30.330 | NA | NA | 3.93 |
| $c d 325285$ | $c d 325285$ | 0 | 36.514 | 1 | 0.49 | 3.61 |
| $c d 357273$ | $c d 357273$ | 1 | 37.811 | 0 | 1.65 | 5.30 |
| $c d 872422$ | $c d 872422$ | 1 | 20.393 | 0 | 1.95 | 4.07 |
| $c d 1005389$ | $c d 1005389$ | 1 | 28.210 | 1 | 0.35 | 3.90 |

... and than check that the genotypes have imported right:

```
> g0 <- as.character(df@gtdata)
> g0
\begin{tabular}{lllllll} 
& rs1001 & rs6679 & rs2401 & rs123 & rs6679 & rs876 \\
cditoA1
\end{tabular}
> as.character(df@gtdata@strand)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
    "+" "+" "+" "-" "-" "+" "-"
> as.character(df@gtdata@coding)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
    "AG" "GT" "CA" "TG" "GT" "TG" "CA"
```

In a real Illumina file, a coding on the TOP strand is supplied. Then, the file will normally look like

| name | chr | pos | "cd289982" | "cd325285" | "cd357273" | "cd872422" | "cd1005389" |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| rs1001 | 1 | 1235 | AA | AG | AG | AA | GG |
| rs6679 | 9 | 2344 | GT | GG | GG | TG | GG |
| rs2401 | 22 | 3455 | AA | CC | CC | CC | AC |
| rs123 | X | 32535 | TT | GT | TT | TT | TT |
| rs6679 | XY | 2344 | GT | GG | GG | TG | GG |
| rs876 | Y | 23556 | 00 | 00 | TT | GG | TT |
| mitoA1 | mt | 24245 | AA | CC | 00 | 00 | 00 |

and the conversion command will be

```
> convert.snp.illumina(inf = "RData/gen0.illuwos", out = "RData/genOiwos.raw",
+ strand = "+")
Reading genotypes from file 'RData/gen0.illuwos' ...
Writing to file 'RData/gen0iwos.raw' ...
... done.
```

In this particular data set, after conversion, the " + " strand will actually mean not "forward", but TOP - something you should remember for this particular data. The resulting file will look like this:

You can load the data with

```
> df <- load.gwaa.data(phe = "RData/pheO.dat", gen = "RData/genOiwos.raw",
+ force = T)
ids loaded...
marker names loaded...
chromosome data loaded...
map data loaded...
```

```
allele coding data loaded...
strand data loaded...
genotype data loaded...
snp.data object created...
assignment of gwaa.data object FORCED; X-errors were not checked!
    Obviously, the "strand" is always "+" (here it means TOP):
> g1 <- as.character(df@gtdata)
> g1
\begin{tabular}{llllllll} 
& rs1001 & rs6679 & rs2401 & rs123 & rs6679 & rs876 & mitoA1 \\
cd289982 & "A/A" & "G/T" & "A/A" & "T/T" & "G/T" & NA & "A/A" \\
cd325285 & "A/G" & "G/G" & "C/C" & "T/G" & "G/G" & NA & "C/C" \\
cd357273 & "A/G" & "G/G" & "C/C" & "T/T" & "G/G" & "T/T" & NA \\
cd872422 & "A/A" & "G/T" & "C/C" & "T/T" & "G/T" & "G/G" & NA \\
cd1005389 & "G/G" & "G/G" & "C/A" & "T/T" & "G/G" & "T/T" & NA
\end{tabular}
> as.character(df@gtdata@strand)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
> as.character(df@gtdata@coding)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
    "AG" "GT" "CA" "TG" "GT" "TG" "CA"
```

We can see that the genotypes are identical to ones we imported previously, as should be the case:

```
> g0 == g1
```

|  | rs1001 | rs6679 | rs2401 | rs123 | rs6679 | rs876 | mitoA1 |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cd289982 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |
| cd325285 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |
| cd357273 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |
| cd872422 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |
| cd1005389 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |

## A. 2 Converting PLINK tped files

PLINK TPED (transposed-pedigree) format bears notable similarity to what we call "Illumina" format, with few exceptions: (1) there is no header line giving field names (and therefore IDs are stored in a separate file) (2) the first column gives chromosome, second - SNP name, third genetic map (usually kept as zeroes), the fourth - physical position, and, starting with the fifth column, genotypic data are listed, (3) finally, within a genotypes, alleles are separated with a space. In TPED format, the data we already worked with would look like

| 1 | rs1001 | 0 | 1235 | A A | A | G | A | G | A | A | G | G |  |
| ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9 | rs6679 | 0 | 2344 | G T | G | G | G | G | T | G | G | G |  |
| 22 | rs2401 | 0 | 3455 | A A | C | C | C | C | C | C | A | C |  |
| X | rs123 | 0 | 32535 | T | T | G | T | T | T | T | T | T | T |
| XY | rs6679 | 0 | 2344 | G T | G | G | G | G | T | G | G | G |  |
| Y | rs876 | 0 | 23556 | 0 | 0 | 0 | 0 | T T | G | G | T | T |  |
| mt | mitoA1 | 0 | 24245 | A A | C | C | 0 | 0 | 0 | 0 | 0 | 0 |  |

Obviously, a separate file is needed to keep correspondence between genotypes and IDs. This file emulated standard pedigree file without a header line. The file, conventionally called a TFAM-file, should contain six columns, corresponding to pedigree ID, ID, father, mother, sex, and affection. Only the second column is used by GenABEL - please make sure you use unique IDs. Consequently, it does not matter what pedigree ID, father/mother, sex, or affection status you assign in the file - the real information is coming from the phenotypic data file. The TFAM file for our data will look like this:

```
cd289982 0 0 1 0
cd325285 0 0 1 0
cd357273 0 0 1 0
1 cd872422 0 0 1 0
1 cd1005389 0 0 1 0
```

You can convert the data from PLINK TPED format to the GenABEL format using command convert.snp.tped:

```
> convert.snp.tped(tped = "RData/gen0.tped", tfam = "RData/gen0.tfam",
+ out = "RData/genOtped.raw", strand = "+")
Reading individual ids from file 'RData/gen0.tfam' ...
... done. Read 5 individual ids from file 'RData/gen0.tfam'
Reading genotypes from file 'RData/gen0.tped' ...
...done. Read 7 SNPs from file 'RData/gen0.tped'
Writing to file 'RData/gen0tped.raw' ...
... done.
```

and load the data with

```
> df <- load.gwaa.data(phe = "RData/phe0.dat", gen = "RData/genOtped.raw",
+ force = T)
ids loaded...
marker names loaded...
chromosome data loaded...
map data loaded...
allele coding data loaded...
strand data loaded...
genotype data loaded...
snp.data object created...
assignment of gwaa.data object FORCED; X-errors were not checked!
```

Obviously, the "strand" is always "+" (meaning TOP):

```
> g1 <- as.character(df@gtdata)
> g1
```

```
    rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
```

    rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
    cd289982 "A/A" "G/T" "A/A" "T/T" "G/T" NA "A/A"
cd289982 "A/A" "G/T" "A/A" "T/T" "G/T" NA "A/A"
cd325285 "A/G" "G/G" "C/C" "T/G" "G/G" NA "C/C"
cd325285 "A/G" "G/G" "C/C" "T/G" "G/G" NA "C/C"
cd357273 "A/G" "G/G" "C/C" "T/T" "G/G" "T/T" NA
cd357273 "A/G" "G/G" "C/C" "T/T" "G/G" "T/T" NA
cd872422 "A/A" "G/T" "C/C" "T/T" "G/T" "G/G" NA
cd872422 "A/A" "G/T" "C/C" "T/T" "G/T" "G/G" NA
cd1005389 "G/G" "G/G" "C/A" "T/T" "G/G" "T/T" NA
cd1005389 "G/G" "G/G" "C/A" "T/T" "G/G" "T/T" NA
> as.character(df@gtdata@strand)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
> as.character(df@gtdata@coding)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
"AG" "GT" "CA" "TG" "GT" "TG" "CA"

```

We can see that the genotypes are identical to ones we imported previously, as should be the case:
```

> g0 == g1

```
\begin{tabular}{lrrrrrrrr} 
& rs1001 & rs6679 & rs2401 & rs123 & rs6679 & rs876 & mitoA1 \\
cd289982 & TRUE & TRUE & TRUE & TRUE & TRUE & NA & TRUE \\
cd325285 & TRUE & TRUE & TRUE & TRUE & TRUE & NA & TRUE \\
cd357273 & TRUE & TRUE & TRUE & TRUE & TRUE & TRUE & NA \\
cd872422 & TRUE & TRUE & TRUE & TRUE & TRUE & TRUE & NA \\
cd1005389 & TRUE & TRUE & TRUE & TRUE & TRUE & TRUE & NA
\end{tabular}

\section*{A. 3 Converting linkage-like files}

Linkage-like files, also known as pre-makeped files, or pedigree files, represent a historic format which dates back to the time when only few markers could be typed - thus the number of subjects was usually greater than the number of markers. In that situation, it was natural and obvious to keep IDs in rows and markers in columns. In the first six columns, standard linkage-like file would contain pedigree ID, ID, father's ID, mother's ID, sex (coded as \(1=\) male and \(2=\) female \()\), and affection status ( \(0=\) unknown, \(1=\) unaffected, \(2=\) affected \()\). In the following columns, genotypic information is provided. Alleles of the same genotype could be separated by a space, or by a slash ("/"). Thus the data we are working with could be presented as
```

1 cd289982 0 0 1 0 A A G T A A T T G T 0 0 A A
1 cd325285 0 0 1 0 A G G G C C G T G G 0 0 C C
1 cd357273 0 0 1 0 A G G G C C T T G G T T 0
1 cd872422 0 O 1 0 A A T G C C T T T G G G 0}
1 cd1005389 0 0 1 0 G G G G A C T T G G T T 0 0

```

As you can see, this file misses header line, and information what are the SNP names, position, etc. should be provided in a separate MAP-file. GenABEL accepts map in Merlin format, and an extended format. A map in Merlin format consist of header line, giving column names, and three columns with chromosome, name and position information, for example:
\begin{tabular}{lll} 
chr & name & pos \\
1 & rs1001 & 1235 \\
9 & rs6679 & 2344 \\
22 & rs2401 & 3455 \\
\(X\) & rs123 & 32535 \\
\(X Y\) & rs6679 & 2344 \\
\(Y\) & rs876 & 23556 \\
mt & mitoA1 & 24245
\end{tabular}

The data can be converted to the internal GenABEL format with
```

> convert.snp.ped(ped = "RData/gen0.ped", map = "RData/map0.dat",

+ out = "RData/gen0pedwos.raw", strand = "+")
Reading map from file 'RData/map0.dat' ...
... done. Read positions of 7 markers from file 'RData/map0.dat'
Reading genotypes from file 'RData/gen0.ped' ...
...done. Read information for 5 people from file 'RData/gen0.ped'
Analysing marker information ...
Writing to file 'RData/gen0pedwos.raw' ...
... done.

```
and loaded with
```

> df <- load.gwaa.data(phe = "RData/phe0.dat", gen = "RData/gen0pedwos.raw",

+ force = T)
ids loaded...
marker names loaded...
chromosome data loaded...
map data loaded...
allele coding data loaded...
strand data loaded...
genotype data loaded...
snp.data object created...
assignment of gwaa.data object FORCED; X-errors were not checked!

```

We can inspect the genotypic data and check that conversion results are identical to previous runs with
```

> g1 <- as.character(df@gtdata)
> g1

|  | rs1001 | rs6679 | rs2401 | rs123 | rs6679 | rs876 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| cd289982 | "A/A" | "G/T" | "A/A" | "T/T" | "G/T" | NA | "A/A"

```
```

> as.character(df@gtdata@strand)
rs1001 rs6679 rs2401 rrs123 rs6679 rs876 mitoA1
> as.character(df@gtdata@coding)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
"AG" "GT" "CA" "TG" "GT" "TG" "CA"
> g0 == g1

|  | rs1001 | rs6679 | rs2401 | rs123 | rs6679 | rs876 | mitoA1 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cd289982 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |
| cd325285 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |
| cd357273 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |
| cd872422 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |
| cd1005389 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |

```

If you are willing to import strand information, you can make use of the extended map format. In this format the strand information is added to the map-file:
\begin{tabular}{lllll} 
chr & name & pos & strand coding \\
1 & rs1001 & 1235 & + & AG \\
9 & rs6679 & 2344 & + & TG \\
22 & rs2401 & 3455 & + & AC \\
X & rs123 & 32535 & - & GT \\
XY & rs6679 & 2344 & - & GT \\
Y & rs876 & 23556 & + & GT \\
mt & mitoA1 & 24245 & - & AC
\end{tabular}

The data can be converted to the internal GenABEL format with
```

> convert.snp.ped(ped = "RData/gen0.ped", map = "RData/emap0.dat",

+ out = "RData/genOped.raw", strand = "file")
Reading map from file 'RData/emap0.dat' ...
... done. Read positions of 7 markers from file 'RData/emap0.dat'
Reading genotypes from file 'RData/gen0.ped' ...
...done. Read information for 5 people from file 'RData/gen0.ped'
Analysing marker information ...
Writing to file 'RData/genOped.raw' ...
... done.

```

Note that option strand==file was used to specify that the extended map format should be used. The data can be loaded with
```

> df <- load.gwaa.data(phe = "RData/pheO.dat", gen = "RData/genOped.raw",

+ force = T)

```
```

ids loaded...
marker names loaded...
chromosome data loaded...
map data loaded...
allele coding data loaded...
strand data loaded...
genotype data loaded...
snp.data object created...
assignment of gwaa.data object FORCED; X-errors were not checked!

```

We can inspect the genotypic data and check that conversion results are identical to previous runs with
```

> g1 <- as.character(df@gtdata)
> g1
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
cd289982 "A/A" "G/T" "A/A" "T/T" "G/T" NA "A/A"
cd325285 "A/G" "G/G" "C/C" "T/G" "G/G" NA "C/C"
cd357273 "A/G" "G/G" "C/C" "T/T" "G/G" "T/T" NA
cd872422 "A/A" "G/T" "C/C" "T/T" "G/T" "G/G" NA
cd1005389 "G/G" "G/G" "C/A" "T/T" "G/G" "T/T" NA
> as.character(df@gtdata@strand)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
"+" "+" "+" "-" "-" "+" "-"
> as.character(df@gtdata@coding)
rs1001 rs6679 rs2401 rs123 rs6679 rs876 mitoA1
"AG" "GT" "CA" "TG" "GT" "TG" "CA"
> g0 == g1

|  |  | rs1001 | rs6679 | rs2401 | rs123 | rs6679 | rs876 | mitoA1 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cd289982 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |  |
| cd325285 | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE |  |
| cd357273 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |  |
| cd872422 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |  |
| cd1005389 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | NA |  |

```

\section*{A. 4 Converting from MACH format}

The data from MACH format can be converted by using convert.snp.mach. This function actually calls convert.snp.ped in specific format. MACH software is widely used for SNP imputations. For our needs we consider two files produced by MACH: pedigree file with (the imputed) genotypic data, and infofile, containing information about quality of imputations for particular SNP.

SEE HELP FOR convert.snp.mach for further details.

\section*{A. 5 Converting from text format}

\section*{Appendix B}

\section*{Answers to exercises}

\section*{B.0.1 Exercise 2.1:}

For the first person id is " p 1 " and sex code is 1 ( \(1=\) male, \(2=\) female)
> srdta@gtdata@idnames[1]
[1] "p1"
> srdta@gtdata@male[1]
p1
1
For the 22 nd person id is "p22" and sex code is 1 :
> srdta@gtdata@idnames [22]
[1] "p22"
> srdta@gtdata@male[22]
p22
1
Among first 100 subjects, there are 53 males:
> sum(srdta@gtdata@male[1:100])
[1] 53
Among 4th hundred subjects there are 45 females:
> 100 - sum(srdta@gtdata@male[301:400])
[1] 45
Male proportion among first 1000 people is
> mean(srdta@gtdata@male[1:1000])
[1] 0.508
Female proportion among second 1000 people is
> 1 - mean(srdta@gtdata@male[1001:2000])
[1] 0.476
Name, chromosome and map position of the 33rd marker are:
> srdta@gtdata@snpnames [33]
[1] "rs422"
> srdta@gtdata@chromosome[33]
rs422
1
Levels: 1
> srdta@gtdata@map[33]
rs422
105500
The map positions for and distance between markers 25 and 26 are:
> pos25 <- srdta@gtdata@map[25]
> pos25
rs365
91250
> pos26 <- srdta@gtdata@map[26]
> pos26
rs372
92750
> pos26 - pos25
rs372
1500

\section*{B.0.2 Exercise 2.2:}

Value of the 4th variable of person 75 :
> srdta@phdata[75, 4]
[1] -0.04
Value for the variable 1 is
> srdta@phdata[75, 1]
[1] "p75"
Also, if we check first 10 elements we see
> srdta@phdata[1:10, 1]
[1] "p1" "p2" "p3" "p4" "p5" "p6" "p7" "p8" "p9" "p10"
This is personal ID.
The sum for variable 2 is
> sum(srdta@phdata[, 2])
[1] 1275
This is sex variable.

\section*{B.0.3 Exercise 2.3:}

To obtain the number of people with age \(>65\) y.o., you can use any of the following
```

> sum(srdta@phdata\$age > 65)

```
[1] 48
```

> vec <- which(srdta@phdata\$age > 65)

```
> length(vec)
[1] 48
To get sex of these people use any of:
```

> sx65 <- srdta@phdata$sex[srdta@phdata$age > 65]
> sx65

```
 [39] 100100000111
```

> sx65 <- srdta@phdata\$sex[vec]
> sx65

```

[39] 100100000111
Thus, number of males is:
\(>\operatorname{sum}(\mathrm{sx} 65)\)
[1] 26
To conclude, the proportion of male is 0.541666666666667
Distribution of qt3 in people younger and older than 65 are:
> summary(srdta@phdata\$qt3[vec])
```

    Min. 1st Qu. Median Mean 3rd Qu. Max.
    0.730
        2.690 3.480
        3.499 4.265
        5.840
    > sd(srdta@phdata\$qt3[vec], na.rm = TRUE)
[1] 1.128701

```
> young <- which(srdta@phdata\$age < 65)
> summary(srdta@phdata\$qt3[young])
    Min. 1st Qu. Median Mean 3rd Qu. Max. NA's
    \(\begin{array}{lllllll}-1.97 & 1.83 & 2.58 & 2.59 & 3.35 & 6.34 & 11.00\end{array}\)
> sd(srdta@phdata\$qt3[young], na.rm = TRUE)
[1] 1.093374

\section*{B.0.4 Exercise 2.3:}
> summary(srdta@phdata\$age)
\begin{tabular}{rrrrrr} 
Min. & 1st Qu. & Median & Mean & 3rd Qu. & Max. \\
24.10 & 45.10 & 50.00 & 50.04 & 54.80 & 71.60
\end{tabular}

The histogram for qt2 looks strange: it seems there are few very strong outliers (figure B.1) You can also see that with summary:
```

> summary(srdta@phdata\$qt2)
Min. 1st Qu. Median Mean 3rd Qu. Max.
0.000 4.220 5.045 6.122 5.910 888.000

```

\section*{B.0.5 Exercise 1:}

How many SNPs are described in this data frame?
> attach (popdat)
The following object(s) are masked from data2@phdata :
sex
> names (popdat)
[1] "subj" "sex" "aff" "qt" "snp1" "snp2" "snp3" "snp4" "snp5"
[10] "snp6" "snp7" "snp8" "snp9" "snp10"
The answer is 10 snps

\section*{Histogram of srdta@phdata\$qt2}


Figure B.1: Histogram of qt2

\section*{B.0.6 Exercise 2:}

What is the frequency (proportion) of snp1 allele A? What is its frequency in these affected (aff==1)?
> summary (snp1)
Number of samples typed: 2374 (95\%)
Allele Frequency: (2 alleles)
Count Proportion
\begin{tabular}{lrr} 
A & 3462 & 0.73 \\
B & 1286 & 0.27 \\
NA & 252 & NA
\end{tabular}

Genotype Frequency:
Count Proportion
\begin{tabular}{lll} 
B/B 199 & 0.08
\end{tabular}

A/B \(888 \quad 0.37\)
A/A \(1287 \quad 0.54\)
NA 126 NA

Heterozygosity (Hu) \(=0.3950646\)
Poly. Inf. Content \(=0.3169762\)

The frequency of A in all subjects is 0.73 .
```

> summary(snp1[aff == 1])
Number of samples typed: 519 (94.5%)
Allele Frequency: (2 alleles)
Count Proportion
A 729 0.7
B 309 0.3
NA 60 NA
Genotype Frequency:
Count Proportion
B/B 48 0.09
A/B 213 0.41
A/A 258 0.50
NA 30 NA
Heterozygosity (Hu) = 0.4185428
Poly. Inf. Content = 0.3307192

```

The frequency of A in affected subjects is 0.7 .

\section*{B.0.7 Exercise 3:}

How many cases and controls are present?
```

> table(aff)
aff
0 1
1 9 5 1 5 4 9

```

There are 549 cases and 1951 controls.

\section*{B.0.8 Exercise 4:}

If all subjects are used to test HWE, are there any SNPs out of HWE at nominal \(P \leq 0.05\) ? Which ones?
```

> HWE.exact(snp1)

```

Exact Test for Hardy-Weinberg Equilibrium
```

data: snp1

```
N11 = 1287, N12 = 888, N22 = 199, N1 = 3462, N2 = 1286, p-value =
0.01083
> HWE.exact (snp10)

Exact Test for Hardy-Weinberg Equilibrium
data: snp10
\(\mathrm{N} 11=1792, \mathrm{~N} 12=552, \mathrm{~N} 22=40, \mathrm{~N} 1=4136, \mathrm{~N} 2=632, \mathrm{p}\)-value \(=0.7897\)
Only SNP 1 is out of HWE in the total sample.

\section*{B.0.9 Exercise 5:}

If only controls are used to test the SNPs which are out of HWE in total sample, are these still out of HWE?
```

> HWE.exact(snp1[aff == 0])
Exact Test for Hardy-Weinberg Equilibrium
data: snp1[aff == 0]
N11 = 1029, N12 = 675, N22 = 151, N1 = 2733, N2 = 977, p-value =
0.008393

```

Yes, SNP 1 is out of HWE also in controls.

\section*{B.0.10 Exercise 6:}

Which SNP pairs are in strong LD \(\left(r^{2} \geq 0.8\right)\) ?
\begin{tabular}{lrrrrrrrrrr}
\(>L D(p o p d a t[, ~ 5: 14]) \$ " R^{\wedge} 2^{\prime \prime}\) \\
& snp1 & snp2 & snp3 & snp4 & snp5 & snp6 & snp7 & snp8 & snp9 & snp10 \\
snp1 & NA & 0.016 & 0.235 & 0.206 & 0.258 & 0.227 & 0.152 & 0.117 & 0.090 & 0.000 \\
snp2 & NA & NA & 0.004 & 0.004 & 0.005 & 0.004 & 0.000 & 0.000 & 0.000 & 0.000 \\
snp3 & NA & NA & NA & 0.602 & 0.457 & 0.346 & 0.641 & 0.031 & 0.042 & 0.001 \\
snp4 & NA & NA & NA & NA & 0.803 & 0.650 & 0.729 & 0.027 & 0.037 & 0.002 \\
snp5 & NA & NA & NA & NA & NA & 0.874 & 0.586 & 0.034 & 0.046 & 0.002 \\
snp6 & NA & NA & NA & NA & NA & NA & 0.670 & 0.030 & 0.040 & 0.002 \\
snp7 & NA & NA & NA & NA & NA & NA & NA & 0.020 & 0.027 & 0.003 \\
snp8 & NA & NA & NA & NA & NA & NA & NA & NA & 0.002 & 0.000 \\
snp9 & NA & NA & NA & NA & NA & NA & NA & NA & NA & 0.001 \\
snp10 & NA & NA & NA & NA & NA & NA & NA & NA & NA & NA
\end{tabular}

SNP pairs 4-5 and 5-6 have \(r^{2} \geq 0.8\).

\section*{B.0.11 Exercise 7:}

For SNPs in strong LD, what is \(r^{2}\) for separate samples of cases and controls?
For controls,
```

> LD(data.frame(snp4, snp5, snp6)[aff == 0, ])\$"R^2"
snp4 snp5 snp6
snp4 NA 0.806591 0.6419715
snp5 NA NA 0.8661005
snp6 NA NA NA

```

For cases,
```

> LD(data.frame(snp4, snp5, snp6)[aff == 1, ])\$"R^2"
snp4 snp5 snp6
snp4 NA 0.7951475 0.6773275
snp5 NA NA 0.9083237
snp6 NA NA NA

```

Note that the fact that LD is higher in cases may mean nothing because the estimates of LD are biased upwards with smaller sample sizes. For example in a small sample ( 5 people) of controls we expect even higher LD because of strong upward bias:
```

> LD(popdat[which(aff == 0)[1:5], 8:10])\$"R^2"
snp4 snp5 snp6
snp4 NA 0.9995876 0.9995876
snp5 NA NA 0.9995876
snp6 NA NA NA

```

More elaborate methods, such as that by Zaykin, are required to contrast LD between sample of unequal size.

\section*{B.0.12 Exercise 8:}

Is there significant association between affection status and sex? What is \(P\) value for association?
```

> glm(aff ~ sex, family = binomial())
Call: glm(formula = aff ~ sex, family = binomial())
Coefficients:
(Intercept) sex
-1.3197 0.1006

```
Degrees of Freedom: 2499 Total (i.e. Null); 2498 Residual
Null Deviance: 2632
Residual Deviance: 2631 AIC: 2635

There is significant ( \(P=0.03\) ) association.

\section*{B.0.13 Exercise 9:}

Is association between the disease and qt significant?
```

> glm(aff ~ qt, family = binomial())
Call: glm(formula = aff ~ qt, family = binomial())
Coefficients:
(Intercept) qt
-1.26769 -0.02514

```
Degrees of Freedom: 2499 Total (i.e. Null); 2498 Residual
Null Deviance: 2632
Residual Deviance: 2632
    AIC: 2636

There is no significant \((P=0.6)\) association.

\section*{B.0.14 Exercise 10:}

Which SNPs are associated with the quantitative trait qt at nominal \(P \leq 0.05\) ?
Use 2 d.f. test.
```

> summary(lm(qt ~ snp1))
Call:
lm(formula = qt ~ snp1)
Residuals:

| Min | 1Q | Median | 3Q | Max |
| ---: | ---: | ---: | ---: | ---: |
| -3.52609 | -0.66427 | -0.01110 | 0.67648 | 3.54622 |

Coefficients:
Estimate Std. Error t value Pr(>|t|)

```
\begin{tabular}{lrrrr} 
(Intercept) & -0.02846 & 0.02758 & -1.032 & 0.3022 \\
snp1A/B & 0.08200 & 0.04316 & 1.900 & 0.0575. \\
snp1B/B & 0.18644 & 0.07536 & 2.474 & \(0.0134 *\)
\end{tabular}


Residual standard error: 0.9893 on 2371 degrees of freedom (126 observations deleted due to missingness)
Multiple R-squared: 0.00335, Adjusted R-squared: 0.002509
F-statistic: 3.985 on 2 and 2371 DF, p-value: 0.01873
> summary(lm(qt ~ snp10))
Call:
lm(formula = qt ~ snp10)

Residuals:
\begin{tabular}{rrrrr} 
Min & 1Q & Median & 3Q & Max \\
-3.586464 & -0.677484 & 0.001935 & 0.673270 & 3.412527 \\
\multicolumn{6}{c}{} \\
Coefficients: \\
Estimate & Std. Error & \(t\) & value & \(\operatorname{Pr}(>|t|)\) \\
(Intercept) & 0.01915 & 0.02344 & 0.817 & 0.414 \\
snp10A/B & 0.01277 & 0.04829 & 0.264 & 0.792 \\
snp10B/B & 0.17178 & 0.15860 & 1.083 & 0.279
\end{tabular}

Residual standard error: 0.9921 on 2381 degrees of freedom
(116 observations deleted due to missingness)
Multiple R-squared: 0.0005072, Adjusted R-squared: -0.0003324
F-statistic: 0.6041 on 2 and 2381 DF, p-value: 0.5467
SNPs \(1,4,5\) an 9 are significantly associated at nominal \(P \leq 0.05\).

\section*{B.0.15 Exercise 11:}

Test each SNP for association with the affection status, using 2 d.f. test. Which SNPs are significantly associated at nominal \(P \leq 0.05\) ? How can you describe the model of action of the significant SNPs?
```

> x <- glm(aff ~ snp5, family = binomial())
> x
Call: glm(formula = aff ~ snp5, family = binomial())
Coefficients:
(Intercept) snp5A/A snp5B/B
Degrees of Freedom: 2382 Total (i.e. Null); 2380 Residual
(117 observations deleted due to missingness)

```
```

Null Deviance: 2440
Residual Deviance: 2431
AIC: 2437
> anova(x, test = "Chisq")
Analysis of Deviance Table
Model: binomial, link: logit
Response: aff
Terms added sequentially (first to last)

|  | Df Deviance Resid. Df | Resid. Dev $P(>\mid$ Chil $)$ |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: |
| NULL |  |  | 2382 | 2440.4 |  |
| snp5 | 2 | 9.2395 | 2380 | 2431.2 | $0.009855 * *$ |

Signif. codes: 0 âĂŸ***âĂŹZ 0.001 âĂŸ**âĀŹŹ 0.01 âĂŸ*âĀŹ
> x <- glm(aff ~ snp10, family = binomial())
> x
Call: glm(formula = aff ~ snp10, family = binomial())
Coefficients:
(Intercept) snp10A/B snp10B/B
-1.3703 0.2909 -0.1803
Degrees of Freedom: 2383 Total (i.e. Null); 2381 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2468 AIC: 2474
> anova(x, test = "Chisq")
Analysis of Deviance Table
Model: binomial, link: logit
Response: aff
Terms added sequentially (first to last)

|  | Df Deviance | Resid. Df | Resid. Dev | P( $>\mid$ Chil) |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| NULL |  | 2383 | 2475.1 |  |  |
| snp10 | 2 | 6.7328 | 2381 | 2468.4 | $0.03451 *$ |

Signif. codes: 0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

```

The SNPs 5 an 10 are significantly associated at \(P \leq 0.05\). The model of action of SNP5 can be ddescribed as recessive (while the risk for AA and AB is not significantly different, there is 1.4 times increased risk for these homozygous for \(\mathrm{BB})\). The SNP 10 demonstrates somewhat weird action with the risk increased in heterozygous AB individuals. However, the confidence interval for BB is large and therefore we can not claim that BB is not increasing the risk.

\section*{B.0.16 Exercise 12:}

For the SNPs selected in previous question, test association using additive model. Which SNPs are still associated?
```

> glm(aff ~ as.numeric(snp5), family = binomial())
Call: glm(formula = aff ~ as.numeric(snp5), family = binomial())
Coefficients:
(Intercept) as.numeric(snp5)
-1.5695 0.1094
Degrees of Freedom: 2382 Total (i.e. Null); 2381 Residual
(117 observations deleted due to missingness)
Null Deviance: 2440
Residual Deviance: 2438 AIC: 2442
> glm(aff ~ as.numeric(snp10), family = binomial())
Call: glm(formula = aff ~ as.numeric(snp10), family = binomial())
Coefficients:
(Intercept) as.numeric(snp10)
-1.5539 0.1976
Degrees of Freedom: 2383 Total (i.e. Null); 2382 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2471 AIC: 2475

```

Only SNP 10 is significantly associated under the additive model.

\section*{B.0.17 Exercise 13:}

If you adjust the analysis under additive model (question 12) for significant covariates which you discovered in questions 8 and 9 , are these findings still significant?
```

> glm(aff ~ sex + snp10, family = binomial())
Call: glm(formula = aff ~ sex + snp10, family = binomial())
Coefficients:
(Intercept) sex snp10A/B snp10B/B

```
```

    -1.41894 0.09513 0.29230 -0.18471
    Degrees of Freedom: 2383 Total (i.e. Null); 2380 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2467 AIC: 2475

```

Yes, SNP 10 becomes even a bit more significantly associated after adjusting for sex.

\section*{B.0.18 Exercise 14:}

Test association between aff and snp5 and snp10, allowing for the SNPs interaction effect. Use arbitrary (not an additive) model. Do you observe significant interaction? How can you describe the model of concert action of snp5 and snp10?
```

> glm(aff ~ snp5 * snp10, family = binomial())
Call: glm(formula = aff ~ snp5 * snp10, family = binomial())
Coefficients:

| (Intercept) | snp5A/A | snp5B/B | snp10A/B |
| ---: | ---: | ---: | ---: |
| -1.50840 | -0.41802 | 0.33441 | -0.01403 |
| snp10B/B | snp5A/A:snp10A/B | snp5B/B:snp10A/B | snp5A/A:snp10B/B |
| -0.14983 | 1.48369 | 0.12989 | 0.82348 |
| B $: \operatorname{snp10B/B}$ |  |  |  |
| -0.28562 |  |  |  |

Degrees of Freedom: 2268 Total (i.e. Null); 2260 Residual
(231 observations deleted due to missingness)
Null Deviance: 2282
Residual Deviance: 2243 AIC: 2261

```

It appears that SNP10 genotype is only relevant in these who are homozygous for the low-risk A allele at the SNP5; in such cases SNP 10 allele B is risk increasing. In these homozygous for SNP 5 A , we observe highly significant increase in risk for heterozygotes for SNP10 and increased (though not significantly) risk for SNP 10 BB .

100

\section*{B.0.19 Exercise 4.1:}
```

> data(srdta)

```

Number of people:
> srdta@gtdata@nids
[1] 2500
Number of males:
```

> sum(srdta@gtdata@male)

```
[1] 1275
Number of females:
```

> srdta@gtdata@nids - sum(srdta@gtdata@male)

```
[1] 1225
... or you could get the same answer like this \(\mathbb{1}^{1}\),
> sum(srdta@gtdata@male == 0 )
[1] 1225
The proportion of males can be computed using above results
> sum(srdta@gtdata@male)/srdta@gtdata@nids
[1] 0.51
or by using mean() function:
> mean(srdta@gtdata@male)
[1] 0.51

\section*{B.0.20 Exercise 4.1:}

The names of markers located after 2,490,000 b.p. are
```

> vec <- (srdta@gtdata@map > 2490000)
> srdta@gtdata@snpnames[vec]
[1] "rs9273" "rs9277" "rs9279" "rs9283"
Between 1,100,000 and 1,105,000 b.p.:
> vec <- (srdta@gtdata@map > 1100000 \& srdta@gtdata@map < 1105000)
> srdta@gtdata@snpnames[vec]
[1] "rs4180" "rs4186" "rs4187"

```

\section*{B.0.21 Exercise 4.2:}

To learn what allele of "rs114" is the reference you need to run
> as.character(srdta@gtdata@coding["rs114"])
<NA>
"AT"

\footnotetext{
\({ }^{1}\) This is something covered later in the section ?? ("??")
}

Here, the first ("A") allele is the reference and thus the second (" T ") is the effective one. Remember that when using as.numeric function to convert the genotypes to human-readable and R -operatable format, the homozygotes for reference will be coded as "0", heterozygotes as " 1 " and the non-reference ("effective") homozygotes will be coded as " 2 ":
```

> table(as.character(srdta@gtdata[, "rs114"]), as.numeric(srdta@gtdata[,

+ "rs114"]))

```
\begin{tabular}{lrrr} 
& 0 & 1 & 2 \\
A/A & 1868 & 0 & 0 \\
A/T & 0 & 491 & 0 \\
T/T & 0 & 0 & 34
\end{tabular}

To compute frequency of the effective allele of SNP "rs114" in total sample, you can go two ways. First, we can try to take a sum of all rs114 genotypes and divide it by twice the number of people:
```

> a <- as.numeric(srdta@gtdata[, "rs114"])
> sum(a)

```
[1] NA
This, however, returns NA, because some of the genotypes are missing. We can deal with this problem by running sum() with the option na.rm=TRUE:
```

> sum(a, na.rm = T)

```
[1] 559
However, now we do not know what was the number of people for whom the genotype was measured!

An easier way would be to compute mean value of rs114 with the mean ( . . , na. rm=TRUE) function and divide it by 2 :
```

> mean(a, na.rm = T)/2

```
[1] 0.116799
To compute frequency of the effective allele of "rs114" in males, you can use
```

> amale <- as.numeric(srdta@gtdata[srdta@phdata\$sex == 1, "rs114"])
> mean(amale, na.rm = T)/2

```
[1] 0.1164216
To compute frequency of the effective allele in females, you can use
```

> afemale <- as.numeric(srdta@gtdata[srdta@phdata\$sex == 0, "rs114"])
> mean(afemale, na.rm = T)/2

```
[1] 0.1171942

Actually, the problem that we do not know how many people are measured, can be easily dealt with. This can be done by using is.na(A) function which returns true when some element of \(A\) is not measured. Thus, the number of people with measured genotype for "rs 114 " is
```

> a <- as.numeric(srdta@gtdata[, "rs114"])
> sum(!is.na(a))

```
[1] 2393
And the allele frequency estimate is
```

> sum(a, na.rm = T)/(2 * sum(!is.na(a)))

```

\section*{[1] 0.116799}
exactly the same as above.
The frequencies of the reference allele are computed very simply as one minus the frequency of the effective allele.

\section*{B.0.22 Exercise 4.3:}

To test for HWE in first 10 SNPs in total sample
```

> summary(srdta@gtdata[, 1:10])

```
\begin{tabular}{lrrrrrrrr} 
& NoMeasured & CallRate & Q.2 & P.11 & P.12 & P. 22 & Pexact & Fmax \\
rs10 & 2384 & 0.9536 & 0.13255034 & 1792 & 552 & 40 & \(7.897327 \mathrm{e}-01\) & -0.006880004 \\
rs18 & 2385 & 0.9540 & 0.28029350 & 1232 & 969 & 184 & \(7.608230 \mathrm{e}-01\) & -0.007017332 \\
rs29 & 2374 & 0.9496 & 0.13774221 & 1763 & 568 & 43 & \(7.955141 \mathrm{e}-01\) & -0.007241148 \\
rs65 & 2378 & 0.9512 & 0.71972246 & 182 & 969 & 1227 & \(6.475412 \mathrm{e}-01\) & -0.010016746 \\
rs73 & 2385 & 0.9540 & 0.01341719 & 2331 & 44 & 10 & \(1.792470 \mathrm{e}-12\) & 0.303150234 \\
rs114 & 2393 & 0.9572 & 0.11679900 & 1868 & 491 & 34 & \(7.663683 \mathrm{e}-01\) & 0.005487764 \\
rs128 & 2391 & 0.9564 & 0.02488499 & 2281 & 101 & 9 & \(9.408599 \mathrm{e}-06\) & 0.129600629 \\
rs130 & 2379 & 0.9516 & 0.69377890 & 222 & 1013 & 1144 & \(9.615127 \mathrm{e}-01\) & -0.002140946 \\
rs143 & 2377 & 0.9508 & 0.47728229 & 655 & 1175 & 547 & \(6.512540 \mathrm{e}-01\) & 0.009313705 \\
rs150 & 2369 & 0.9476 & 0.65998312 & 267 & 1077 & 1025 & \(5.518478 \mathrm{e}-01\) & -0.012948436 \\
& Plrt & Chromosome & & & & & &
\end{tabular}
rs10 7.355343e-01 1
rs18 7.315304e-01 1
rs29 7.227853e-01 1
rs65 6.246577e-01 1
rs73 1.281239e-12 1
rs114 7.894076e-01 1
rs128 1.000431e-05 1
rs130 9.168114e-01 1
rs143 6.497695e-01 1
rs150 5.281254e-01 1

To test it in cases
```

> summary(srdta@gtdata[srdta@phdata\$bt == 1, 1:10])

```

in controls
> summary (srdta@gtdata[srdta@phdata\$bt == 0, 1:10])


\section*{B.0.23 Exercise 37:}

Perform meta-analysis of the data presented in table 11.2. Which allele is the risk one? Is this risk significant? What is pooled Odds Ratio and \(95 \%\) confidence interval? Do analysis using at least two methods. Which method is better (best) in this situation? Why?

We first need to unify Odds Ratios by using the same effective allele. Let that be the "risk" allele, as may be guessed from a glance to the data, namely "Pro". When the effects are reported for the other, "Ala" allele, the corresponding ORs for the "Pro" allele can be found using simple relation \(O R_{\text {Pro }}=1 / O R_{\text {Ala }}\).

Thus, the vector of Odds Ratios for "Pro" allele is
```

> or.pro <- c(1/0.67, 0.93, 1.08, 1/0.83, 1.22, 1.23)
> or.pro

```
[1] 1.4925370 .9300001 .0800001 .2048191 .2200001 .230000
The corresponding \(P\) - values are
```

> p <- c(0.013, 0.6, 0.84, 0.4, 0.25, 0.07)

```

Let us find log-ORs
```

> logor.pro <- log(or.pro)
> logor.pro
[1] 0.40047757 -0.07257069 0.07696104 0.18632958

```

Corresponding squared standard errors are
```

> s2 <- logor.pro * logor.pro/qchisq(1 - p, 1)
> s2

```
[1] 0.025997640 .019151210 .145310600 .049015140 .029881020 .01305349
and weights are
```

> w <- 1/s2
> w
[1] $38.4650352 .21601 \quad 6.8818120 .4018633 .4660676 .60788$

```

Thus the pooled estimate of log-OR is
```

> p.logor.pro <- sum(w * logor.pro)/sum(w)
> p.logor.pro

```
[1] 0.1686548
and the standard error is
```

> p.s <- 1/sqrt(sum(w))
> p.s

```
[1] 0.066221

Thus the pooled estimate of Odds Ratio from association between type 2 diabetes and "Ala" allele is
> exp(p.logor.pro)
[1] 1.183711
and the \(95 \%\) confidence interval is
```

> exp(p.logor.pro - 1.96 * p.s)
[1] 1.039627
> exp(p.logor.pro + 1.96 * p.s)
[1] 1.347765

```

The \(\chi^{2}\) test for association and corresponding \(P-\) value are
> p.chi2 <- (p.logor.pro/p.s) ^2
> p.chi2
[1] 6.486429
> p.pval <- 1 - pchisq(p.chi2, 1)
> p.pval
[1] 0.01087011
Z-score pooling though may be more appropriate method for such differentially designed studies (e.g. control groups are very different). To get Z-score pooling working, we need first find Z-scores from P-values
\(>p<-c(0.013,0.6,0.84,0.4,0.25,0.07)\)
\(>z<-\operatorname{sqrt}(q \operatorname{chisq}(1-p, 1))\)
\(>z\)
[1] 2.48376930 .52440050 .20189350 .84162121 .15034941 .8119107
and assign the right sign (let " + " is for the risk effect of "Pro").
```

> effsig <- c(1, -1, 1, 1, 1, 1)
> z <- z * effsig
> z
[1] 2.4837693-0.5244005 0.2018935 0.8416212 1.1503494 1.8119107

```

Now, we need to assign weights to the studies as
```

> n <- c(221, 306, 71, 164, 242, 471)
> w <- sqrt(n)

```
ane the pooled estimate of Z and corresponding \(P-\) value are
```

> zpoo <- sum(w * z)/sqrt(sum(w^2))
> zpoo

```
[1] 2.537333
> 1 - pchisq(zpoo * zpoo, 1)
[1] 0.01117008
As you can see the results are almost identical to the previous obtained with inverse variance pooling.

\section*{B.0.24 Exercise 38:}

Perform meta-analsys excluding the original report (study 1). Is there still significant association between Pro12Ala and diabetes?

The answer to this exercise can be obtained in exactly the same manner, as for the previous one, limiting our consideration to the last five studies.

Thus, the vector of Odds Ratios for "Pro" allele is
```

> or.pro <-c(0.93,1.08,1/0.83,1.22,1.23)
> or.pro
[1] 0.930000 1.080000 1.204819 1.220000 1.230000

```

The corresponding \(P\)-values are
```

>p<-c(0.6, 0.84, 0.4, 0.25, 0.07)

```

Let us find log-ORs
```

> logor.pro <- log(or.pro)
> logor.pro
[1] -0.07257069 0.07696104 0.18632958 0.19885086 0.20701417

```

Corresponding squared standard errors are
```

> s2 <- logor.pro * logor.pro/qchisq(1 - p, 1)
> s2
[1] 0.01915121 0.14531060 0.04901514 0.02988102 0.01305349

```
and weights are
```

> W <- 1/s2
> W
[1] $52.21601 \quad 6.8818120 .4018633 .4660676 .60788$

```

Thus the pooled estimate of log-OR is
```

> p.logor.pro <- sum(w * logor.pro)/sum(w)
> p.logor.pro

```
[1] 0.1216172
and the standard error is
```

> p.s <- 1/squt(sum(W))

```
\(>\) p.s.
[1] 0.07262917
Thus the pooled estimate of Odds Ratio from association between type 2 diabetes and "Ala" allele is
> exp(p.logor.pro)
[1] 1.129322
and the \(95 \%\) confidence interval is
> exp(p.logor.pro - 1.96 * p.s)
[1] 0.9794776
> exp(p.logor.pro + 1.96 * p.s)
[1] 1.302090
The \(\chi^{2}\) test for association and corresponding \(P\)-value are
> p.chi2 <- (p.logor.pro/p.s)~2
> p.chi2
[1] 2.803937
> p.pval <- 1 - pchisq(p.chi2, 1)
> p.pval
[1] 0.09403318
Z-score pooling though may be more appropriate method for such differentially designed studies (e.g. control groups are very different). To get Z-score pooling working, we need first find Z-scores from P-values
```

>p<-c(0.6, 0.84, 0.4, 0.25, 0.07)
> z <- sqrt(qchisq(1 - p, 1))
> z

```
[1] 0.52440050 .20189350 .84162121 .15034941 .8119107
and assign the right sign (let " + " is for the risk effect of "Pro").
```

> effsig <- c(-1, 1, 1, 1, 1)
> z <- z * effsig
> z
[1] $-0.5244005 \quad 0.2018935 \quad 0.8416212 \quad 1.1503494 \quad 1.8119107$

```

Now, we need to assign weights to the studies as
```

> n <- c(306, 71, 164, 242, 471)

```
> w <- sqrt(n)
ane the pooled estimate of Z and corresponding \(P-\) value are
```

> zpoo <- sum(w * z)/sqrt(sum(w`2))
> zpoo
[1] 1.709151

```
> 1 - pchisq(zpoo * zpoo, 1)
[1] 0.08742297
As you can see the results are almost identical to the previous obtained with inverse variance pooling.

\section*{Appendix C}

\section*{Unsorted answers to exercises section 3}

\section*{Answer of Exercise 1}

A-ba-ba!

\section*{Answer of Exercise 2}

Try help.search("Fisher") and help.search("Student t-test"). You will find that the corresponding functions are fisher.test t.test.
Q. 1 : How many SNPs are described in this data frame?
> attach (popdat)

The following object(s) are masked from popdat ( position 3 ) :
aff qt sex \(\operatorname{snp} 1\) snp10 \(\operatorname{snp} 2\) snp3 \(\operatorname{snp} 4\) snp5 \(\operatorname{snp} 6\) snp7 \(\operatorname{snp} 8\) snp9 subj

The following object(s) are masked from data2@phdata :
sex
> names (popdat)
[1] "subj" "sex" "aff" "qt" "snp1" "snp2" "snp3" "snp4" "snp5"
[10] "snp6" "snp7" "snp8" "snp9" "snp10"

The answer is 10 snps
Q. 2 : What is the frequency (proportion) of snp1 allele A? What is its frequency in these affected (aff==1)?
> summary (snp1)


The frequency of A in all subjects is 0.73 .
```

> summary(snp1[aff == 1])
Number of samples typed: 519 (94.5%)
Allele Frequency: (2 alleles)
Count Proportion

| A | 729 | 0.7 |
| :--- | ---: | ---: |
| B | 309 | 0.3 |
| NA | 60 | NA |

```
Genotype Frequency:
            Count Proportion
\begin{tabular}{lll} 
B/B & 48 & 0.09
\end{tabular}
A/B \(\quad 213 \quad 0.41\)
A/A \(258 \quad 0.50\)
NA 30 NA

Heterozygosity (Hu) \(=0.4185428\)
Poly. Inf. Content \(=0.3307192\)
The frequency of A in affected subjects is 0.7 .
Q \(\sqrt[3]{\text { : How many cases and controls are present? }}\)
```

> table(aff)
aff
0 1
1951549

```

There are 549 cases and 1951 controls.
Q. 4 : If all subjects are used to test HWE, are there any SNPs out of HWE at nominal \(P \leq 0.05\) ? Which ones?
```

> HWE. exact(snp1)

```

Exact Test for Hardy-Weinberg Equilibrium
```

data: snp1

```
N11 = 1287, \(\mathrm{N} 12=888\), \(\mathrm{N} 22=199, \mathrm{~N} 1=3462\), \(\mathrm{N} 2=1286\), p-value \(=\)
0.01083
> HWE.exact (snp10)
    Exact Test for Hardy-Weinberg Equilibrium
data: snp10
N11 = 1792, N12 = 552, N22 = 40, N1 = 4136, N2 = 632, p-value = 0.7897

Only SNP 1 is out of HWE in the total sample.
Q.5 : If only controls are used to test the SNPs which are out of HWE in total sample, are these still out of HWE?
> HWE.exact(snp1[aff == 0])
Exact Test for Hardy-Weinberg Equilibrium
data: \(\operatorname{snp1}[\) aff \(==0]\)
\(\mathrm{N} 11=1029, \mathrm{~N} 12=675, \mathrm{~N} 22=151, \mathrm{~N} 1=2733, \mathrm{~N} 2=977, \mathrm{p}\)-value \(=\) 0.008393

Yes, SNP 1 is out of HWE also in controls.
Q. 6 : Which SNP pairs are in strong LD \(\left(r^{2} \geq 0.8\right)\) ?
> LD(popdat[, 5:14])\$"R^2"
\begin{tabular}{lrrrrrrrrrr} 
& snp1 & snp2 & snp3 & snp4 & snp5 & snp6 & snp7 & snp8 & snp9 & snp10 \\
snp1 & NA & 0.016 & 0.235 & 0.206 & 0.258 & 0.227 & 0.152 & 0.117 & 0.090 & 0.000 \\
snp2 & NA & NA & 0.004 & 0.004 & 0.005 & 0.004 & 0.000 & 0.000 & 0.000 & 0.000 \\
snp3 & NA & NA & NA & 0.602 & 0.457 & 0.346 & 0.641 & 0.031 & 0.042 & 0.001 \\
snp4 & NA & NA & NA & NA & 0.803 & 0.650 & 0.729 & 0.027 & 0.037 & 0.002 \\
snp5 & NA & NA & NA & NA & NA & 0.874 & 0.586 & 0.034 & 0.046 & 0.002 \\
snp6 & NA & NA & NA & NA & NA & NA & 0.670 & 0.030 & 0.040 & 0.002 \\
snp7 & NA & NA & NA & NA & NA & NA & NA & 0.020 & 0.027 & 0.003 \\
snp8 & NA & NA & NA & NA & NA & NA & NA & NA & 0.002 & 0.000 \\
snp9 & NA & NA & NA & NA & NA & NA & NA & NA & NA & 0.001 \\
snp10 & NA & NA & NA & NA & NA & NA & NA & NA & NA & NA
\end{tabular}

SNP pairs 4-5 and 5-6 have \(r^{2} \geq 0.8\).
Q.7 : For SNPs in strong LD, what is \(r^{2}\) for separate samples of cases and controls?
For controls,
```

> LD(data.frame(snp4, snp5, snp6)[aff == 0, ])\$"R^2"
snp4 snp5 snp6
snp4 NA 0.806591 0.6419715
snp5 NA NA 0.8661005
snp6 NA NA NA

```

For cases,
```

> LD(data.frame(snp4, snp5, snp6)[aff == 1, ])\$"R`2"
snp4 snp5 snp6
snp4 NA 0.7951475 0.6773275
snp5 NA NA 0.9083237
snp6 NA NA NA

```

Note that the fact that LD is higher in cases may mean nothing because the estimates of LD are biased upwards with smaller sample sizes. For example in a small sample ( 5 people) of controls we expect even higher LD because of strong upward bias:
```

> LD(popdat[which(aff == 0)[1:5], 8:10])\$"R^2"
snp4 snp5 snp6
snp4 NA 0.9995876 0.9995876
snp5 NA NA 0.9995876
snp6 NA NA NA

```

More elaborate methods, such as that by Zaykin, are required to contrast LD between sample of unequal size.
Q.8 : Is there significant association between affection status and sex? What is \(P\)-value for association?
```

> glm(aff ~ sex, family = binomial())
Call: glm(formula = aff ~ sex, family = binomial())
Coefficients:
(Intercept) }r\mathrm{ sex

```
Degrees of Freedom: 2499 Total (i.e. Null); 2498 Residual
Null Deviance:
    2632
Residual Deviance: 2631 AIC: 2635

There is significant \((P=0.03)\) association.
Q, 9 : Is association between the disease and qt significant?
```

> glm(aff ~ qt, family = binomial())
Call: glm(formula = aff ~ qt, family = binomial())
Coefficients:
(Intercept) qt
-1.26769 -0.02514

```
Degrees of Freedom: 2499 Total (i.e. Null); 2498 Residual
Null Deviance: 2632
Residual Deviance: 2632 AIC: 2636

There is no significant ( \(P=0.6\) ) association.
Q. 10 : Which SNPs are associated with the quantitative trait qt at nominal \(P \leq 0.05\) ? Use 2 d.f. test.
> summary(lm(qt ~ snp1))
Call:
\(\operatorname{lm}\left(\right.\) formula \(\left.=q t{ }^{\sim} \operatorname{snp} 1\right)\)
Residuals:
\begin{tabular}{rrrrr} 
Min & \(1 Q\) & Median & 3Q & Max \\
-3.52609 & -0.66427 & -0.01110 & 0.67648 & 3.54622
\end{tabular}

Coefficients:
Estimate Std. Error \(t\) value \(\operatorname{Pr}(>|t|)\)
(Intercept) -0.02846 0.02758 -1.032 0.3022
\(\begin{array}{lllll}\operatorname{snp} 1 \mathrm{~A} / \mathrm{B} & 0.08200 & 0.04316 & 1.900 & 0.0575\end{array}\).
\(\operatorname{snp1B/B} 0.18644 \quad 0.07536 \quad 2.474 \quad 0.0134 *\)
---

Residual standard error: 0.9893 on 2371 degrees of freedom
(126 observations deleted due to missingness)
Multiple R-squared: 0.00335, Adjusted R-squared: 0.002509
F-statistic: 3.985 on 2 and 2371 DF, p-value: 0.01873
> \(\operatorname{summary}\left(\operatorname{lm}\left(q t \sim^{\sim} \operatorname{snp} 10\right)\right)\)
Call:
\(\operatorname{lm}(f o r m u l a=q t \quad \sim \operatorname{snp} 10)\)
Residuals:
\begin{tabular}{rrrrr} 
Min & 1Q & Median & 3Q & Max \\
-3.586464 & -0.677484 & 0.001935 & 0.673270 & 3.412527
\end{tabular}

Coefficients:
Estimate Std. Error \(t\) value \(\operatorname{Pr}(>|t|)\)

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\begin{tabular}{lllll} 
(Intercept) & 0.01915 & 0.02344 & 0.817 & 0.414 \\
snp10A/B & 0.01277 & 0.04829 & 0.264 & 0.792 \\
snp10B/B & 0.17178 & 0.15860 & 1.083 & 0.279
\end{tabular}

Residual standard error: 0.9921 on 2381 degrees of freedom (116 observations deleted due to missingness)
Multiple R-squared: 0.0005072, Adjusted R-squared: -0.0003324
F-statistic: 0.6041 on 2 and 2381 DF, p-value: 0.5467

SNPs \(1,4,5\) an 9 are significantly associated at nominal \(P \leq 0.05\).
Q.11: Test each SNP for association with the affection status, using 2 d.f. test. Which SNPs are significantly associated at nominal \(P \leq 0.05\) ? How can you describe the model of action of the significant SNPs?
```

> x <- glm(aff ~ snp5, family = binomial())
> x
Call: glm(formula = aff ~ snp5, family = binomial())
Coefficients:
(Intercept) snp5A/A snp5B/B
-1.4868 0.2112 0.3387
Degrees of Freedom: 2382 Total (i.e. Null); 2380 Residual
(117 observations deleted due to missingness)
Null Deviance: 2440
Residual Deviance: 2431 AIC: 2437
> anova(x, test = "Chisq")
Analysis of Deviance Table
Model: binomial, link: logit
Response: aff
Terms added sequentially (first to last)

```
```

            Df Deviance Resid. Df Resid. Dev P(>|Chi|)
    NULL 2382 2440.4
snp5 2 9.2395 2380 2431.2 0.009855 **
Signif. codes: 0 âĂŸ̈***âĂŹ 0.001 âăŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ

```
> \(x\) <- glm(aff ~ snp10, family = binomial())
\(>x\)
```

Call: glm(formula = aff ~ snp10, family = binomial())
Coefficients:
(Intercept) }\quad\mathrm{ snp10A/B }\quad\mathrm{ snp10B/B
Degrees of Freedom: 2383 Total (i.e. Null); 2381 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2468 AIC: 2474
> anova(x, test = "Chisq")
Analysis of Deviance Table
Model: binomial, link: logit
Response: aff
Terms added sequentially (first to last)

```
```

        Df Deviance Resid. Df Resid. Dev P(>|Chi|)
    ```
        Df Deviance Resid. Df Resid. Dev P(>|Chi|)
NULL 
```

NULL

```



The SNPs 5 an 10 are significantly associated at \(P \leq 0.05\). The model of action of SNP5 can be ddescribed as recessive (while the risk for AA and AB is not significantly different, there is 1.4 times increased risk for these homozygous for BB ). The SNP 10 demonstrates somewhat weird action with the risk increased in heterozygous AB individuals. However, the confidence interval for BB is large and therefore we can not claim that BB is not increasing the risk.
Q.12 : For the SNPs selected in previous question, test association using additive model. Which SNPs are still associated?
```

> glm(aff ~ as.numeric(snp5), family = binomial())
Call: glm(formula = aff ~ as.numeric(snp5), family = binomial())
Coefficients:
(Intercept) as.numeric(snp5)
-1.5695 0.1094
Degrees of Freedom: 2382 Total (i.e. Null); 2381 Residual
(117 observations deleted due to missingness)
Null Deviance: 2440
Residual Deviance: 2438 AIC: }244

```

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```

> glm(aff ~ as.numeric(snp10), family = binomial())
Call: glm(formula = aff ~ as.numeric(snp10), family = binomial())
Coefficients:
(Intercept) as.numeric(snp10)
-1.5539 0.1976
Degrees of Freedom: 2383 Total (i.e. Null); 2382 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2471 AIC: 2475

```

Only SNP 10 is significantly associated under the additive model.
Q. \(\mathbf{1 3}\) : If you adjust the analysis under additive model (question 12) for significant covariates which you discovered in questions 8 and 9 , are these findings still significant?
```

> glm(aff ~ sex + snp10, family = binomial())
Call: glm(formula = aff ~ sex + snp10, family = binomial())
Coefficients:

| (Intercept) | sex | snp10A/B |
| ---: | ---: | ---: |
| -1.41894 | 0.09513 | snp10B/B |

Degrees of Freedom: 2383 Total (i.e. Null); 2380 Residual
(116 observations deleted due to missingness)
Null Deviance: 2475
Residual Deviance: 2467 AIC: 2475

```

Yes, SNP 10 becomes even a bit more significantly associated after adjusting for sex.
Q.14: Test association between aff and snp5 and snp10, allowing for the SNPs interaction effect. Use arbitrary (not an additive) model. Do you observe significant interaction? How can you describe the model of concert action of snp5 and snp10?
```

> glm(aff ~ snp5 * snp10, family = binomial())
Call: glm(formula = aff ~ snp5 * snp10, family = binomial())
Coefficients:
(Inter
-1
snp10B/B snp5A/A:snp10A/B snp5B/B:snp10A/B snp5A/A:snp10B/B
-0.14983 1.48369 0.12989 0.82348
snp5B/B:snp10B/B
-0.28562

```
```

Degrees of Freedom: 2268 Total (i.e. Null); 2260 Residual
(231 observations deleted due to missingness)
Null Deviance:
2282
Residual Deviance: 2243
AIC: 2261

```

It appears that SNP10 genotype is only relevant in these who are homozygous for the low-risk A allele at the SNP5; in such cases SNP 10 allele B is risk increasing. In these homozygous for SNP 5 A , we observe highly significant increase in risk for heterozygotes for SNP10 and increased (though not significantly) risk for SNP 10 BB .

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[^0]:    ${ }^{1}$ For complete list of arithmetic operations try help("+")

[^1]:    2 Actually, you can apply it to a longer vector too, and then the logical vector will be "expanded" to total length by repeating the original vector head-to-tail. However, we will not use this in our exercises.
    ${ }^{3}$ Because it treats NAs for you

[^2]:    ${ }^{4}$ This may also be true for matrices; more fundamental difference is though that a matrix always contains variables of the same data type, e.g. character or numeric, while a data frame may contain variables of different types

[^3]:    ${ }^{5}$ and also a matrix

[^4]:    ${ }^{6}$ The same argument works for a number of $R$ statistical functions such as mean, median, var, etc

[^5]:    ${ }^{7}$ A hidden trick here is that arithmetic operations treat TRUE as one and FALSE as zero

[^6]:    ${ }^{1}$ There is alternative, top/bottom, strand designation, which does not depend on genomic build. However, it is not always used.

[^7]:    ${ }^{2}$ Ties are generated by the subjects with exactly the same trait values

