ANISOTROPY PRESERVING INTERPOLATION OF DIFFUSION TENSORS

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Diffusion Tensor Images

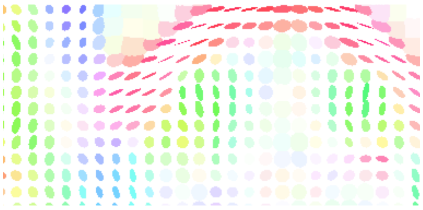

Fig: Zoom on a slice of a Diffusion Tensor Image

Each voxel of the image contains a symmetric positive definite matrix. The principal characteristics of tensors are [1]

- **mean diffusivity** (characterizes the overall mean squared displacement of molecules)
- main **direction** of diffusivities (orientation in space of the structures of the brain)
- degree of **anisotropy** (describes the degree to which molecular displacements vary in space)

The processing of diffusion tensor images requires the development of methods adapted to 'matrix-valued' images. One of the most used method is the Log-Euclidean framework [2], for which

$$d_{\mathrm{LE}}(\mathbf{S}_1,\mathbf{S}_2) = \| \log(\mathbf{S}_1) - \log(\mathbf{S}_2) \|$$

Proposed similarity measure: based on the spectral decomposition

In this work we propose to use the **spectral decomposition** of diffusion tensors in order to preserve all the diffusion information. Let $\mathbf{S}_1$ and $\mathbf{S}_2$ be two diffusion tensors, whose spectral decompositions are $\mathbf{S}_i = \mathbf{U}_i \Lambda_i \mathbf{U}_i^T$, the proposed similarity measure is

$$d_{\mathrm{SD}}^2(\mathbf{S}_1,\mathbf{S}_2) = k(\Lambda_1,\Lambda_2)\, d_{\mathrm{SO}(3)}^2(\mathbf{U}_1,\mathbf{U}_2) + d_{\mathrm{D}^+(3)}^2(\Lambda_1,\Lambda_2)$$

Spherical or isotropic tensors have low orientational information, so their orientation contribution to the metric should be minimal.

$$k \in [0,1]$$

k is constructed to be small (around zero) when any of the tensor is isotropic and close to 1 when both are highly anisotropic.

Distance between orientations is computed through the use of unit quaternions. This parametrization of the rotations enables to speed up the computations.
Because of the non-uniqueness of the spectral decomposition, 4 different rotation matrices represent a single tensor. This implies 8 quaternions for a tensor. The distance between them is computed through

$$d(\mathcal{Q}_1,\mathcal{Q}_2) = \min_{q_2 \in \mathcal{Q}_2} \| q_1^r - q_2 \|$$

This is called the **realignment** of quaternions.

Eigenvalues of tensors are manipulated as **geometric entities** (*i.e.* the logarithms of eigenvalues are used for computations).
The distance between the intensities of diffusion is thus given by

$$d(\Lambda_1,\Lambda_2) = \sqrt{\sum_i \log^2\left(\frac{\lambda_{1,i}}{\lambda_{2,i}}\right)}$$

Mean of a group of N tensors

The mean of a group of *N* tensors is given by

$$\mathbf{S}_\mu = \mathbf{U}_\mu \Lambda_\mu \mathbf{U}_\mu^T$$

$$\Lambda_\mu$$

The eigenvalues of the mean are given by the **geometric mean** of the eigenvalues.

$$\Lambda_\mu = \exp\left(\sum_{i=1}^{N} w_i \log(\Lambda_i)\right)$$

Knowing the mean eigenvalues, the factor *k* can be computed for each tensor.

$$k_i = k(\Lambda_\mu, \Lambda_i)$$

$$\mathrm{U}_\mu$$

The orientation of the mean is given by the mean quaternion.

The **most informative tensor** will be chosen as a **reference** for the computation of the mean. Each quaternion has to be realigned with the reference, because of the non uniqueness of the spectral decomposition.

Most informative tensor: the one for which the product $w_i k_i$ is the maximum.

$$\mathbf{q}_m = \frac{\sum_{i=1}^{N} w_i k_i \mathbf{q}_{i,r}}{\sum_i k_i} \qquad \mathbf{q}_\mu = \frac{\mathbf{q}_m}{\| \mathbf{q}_m \|}$$

This method ensures the **commutativity** and the **uniqueness** of the mean.

Interpolation of diffusion tensors

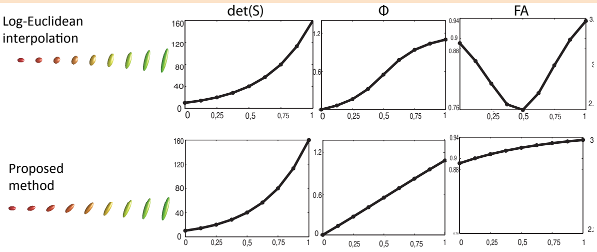

Fig: Geodesic interpolations between two tensors and variation of several characteristics (determinant, angle between the principal directions of diffusion, and fractional anisotropy) of tensors through these interpolations.

Clear differences can be seen between the two methods, namely the main tensor orientation and the degrees of anisotropy. These characteristics are well preserved by the developed framework, which is advantageous for any imaging or tractographic applications.

One example of application of the proposed framework can be found in [3].

[1] Le Bihan, D. (2001), 'Diffusion tensor imaging: Concepts and applications', Journal of Magnetic Resonance Imaging, vol. 13, no. 4, pp. 534-546

[2] Arsigny V. (2006), 'Log-Euclidean metrics for fast and simple calculus on diffusion tensors', Magnetic Resonance in Medicine, vol. 56, no. 2, pp. 411-421

[3] André, E. , 'Iterative method for motion correction of diffusion weighted images', OHBM 2012, Poster #615 MT.

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